

ASYMPTOTIC STABILIZATION OF COMPOSITE PLATES UNDER FLUID LOADING*

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Abstract– This paper presents a solution to the boundary stabilization of a vibrating composite plate under fluid loading. The fluid is considered to be compressible, barotropic and inviscid. A linear control law is constructed to suppress the composite plate vibration. The control forces and moments consist of feedbacks of the velocity and normal derivative of the velocity at the boundaries of the composite plate. The novel features of the proposed method are that (1) it asymptotically stabilizes vibrations of a composite plate in contact with fluid (the fluid has a free surface) via boundary control and without truncation of the model; and (2) the stabilization of both the composite plate vibrations and fluid motion are simultaneously achieved by using only a linear feedback from the composite plate boundaries.

Keywords– Semigroups of operators, LaSalle invariant set theorem, asymptotic stabilization, symmetric angle-ply composite plate, compressible Newtonian Barotropic fluid

1. INTRODUCTION

Today, in many high-tech and industrial processes composite materials such as composite rubber tubes, composite beams, plates and composite shells are widely utilized. They are used in numerous applications such as airplane wings, satellite antennas, bioengineering tissues, flexible manipulators, etc. Aeronautics and aerospace are the most important fields of engineering where the use of composite materials is a common practice. The suitability for this wide range of applications is due to their excellent properties such as being light weight, having high tolerance when subjected to high temperature conditions and also their strength under very heavy loadings.

In this paper, the problem of vibration stabilization of a symmetric angle-ply composite plate in contact with a fluid is considered. Symmetric angle-ply composite plates include broad types of plates, such as Kirchhoff plates, symmetric laminates with multiple generally orthotropic layers, symmetric laminates with multiple anisotropic layers, etc. In most of the recent high-technology applications, composite materials are in contact with a fluid such as air. This leads to a fluid-structure interaction problem.

For fluid-structure systems, the vibration of the structure in contact with a fluid has been thoroughly analyzed by many authors [1-3]. Such problems appear frequently in practice, for example, when studying the veins, pulmonary passages and urinary systems which can be modeled as shells conveying fluid, aero-elastic instabilities around flexible aircraft, container conveying the fluids and dams [1-6].

One of the most challenging practical difficulties which is present in many of the fluid-structure applications is the vibration of the structures. This may be due to relatively low rigidity and small structural damping of structures; where a little excitation may lead to long vibration decay time. Vibration

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of flexible structure is capable of disturbance, discomfort, damage and destruction. In particular, many researchers have studied the problem of vibration suppression (stabilization) of structures (without and with being in contact with a fluid) since a plate is a necessary element in many applications such as the aircraft's skin and flexible structures. In particular, it is widely used in fluid-structure systems [1, 2, 4 and 5].

Boundary stabilization methods are efficient methods to exclude the problems of both control spillover and in-domain measurement and actuation. The boundary actuators designed for the nondiscretized PDE models are often simple compensators which ensure closed-loop stability for an infinite number of modes [7]. Several researchers have proposed boundary actuators for a variety of flexible systems [7-9, 10-12].

Some researchers have been concerned with the fluid-structure stabilization problems, [3 and 13]. In these studies, the fluid has no free surface; however, in fact, in most fluid-structure problems such as dams, large containers, the fluid has at least a free surface. In addition, none of the aforementioned works considered composite plates in their structures. Therefore, in this work, we study the stabilization problem of vibrating composite plate in contact with a fluid having free surface. The fluid is considered to be barotropic compressible Newtonian fluid whereas the plate is taken as symmetric angle-ply composite plate. The required mathematical background includes semigroup techniques and LaSalle's invariant set theorem. We use the semigroup techniques to demonstrate well-posedness of the system. Then benefitting from the Lyapunov stability method and LaSalle's invariant set theorem, we prove the asymptotic stability of the closed loop system.

The remainder of this article is arranged as follows: in section 2, the dynamics of a composite plate and surrounding fluid are presented. Section 3 is devoted to well-posedness and boundary stabilization proof of the fluid-structure problem. Section 4 is devoted to the conclusion.

2. GOVERNING EQUATIONS OF MOTION

a) Fluid Domain

The governing equation for the Newtonian barotropic fluid with low velocity can be simplified from the Navier-Stokes equation to the wave equation [14]. The related equations are listed below,

$$\begin{cases} c^2 \Delta \phi = \phi_{,tt} & \text{in } \Theta \\ \rho_0 \phi_{,t} = -p(x, y, 0, t) & \text{in } \Omega \\ \rho_0 \phi_{,tt} + \rho_0 g \phi_{,n} + p_{e,t} = 0 & \text{in } \Omega_2 \\ \phi_{,n} = 0 & \text{in } \Omega_3 \end{cases} \quad (1)$$

where ϕ is the fluid velocity potential, Ω , Ω_2 and Ω_3 are defined as follows

1) The wet surface or the fluid structure interface, Ω : See Fig. 1,

This is the most essential part of the fluid boundary. The motion of the structure and the normal component of the fluid motion coincide, that is, [14]:

$$\mathbf{v}_f \cdot \mathbf{n} = \mathbf{v}_s \cdot \mathbf{n} \quad (2)$$

where \mathbf{v}_f is the fluid velocity and \mathbf{v}_s is the structure velocity.

In this boundary the following equation can be attained, [14]:

$$\rho_0 \phi_{,t} = -p(x, y, 0, t) \quad (3)$$

where p is the hydrodynamic pressure.

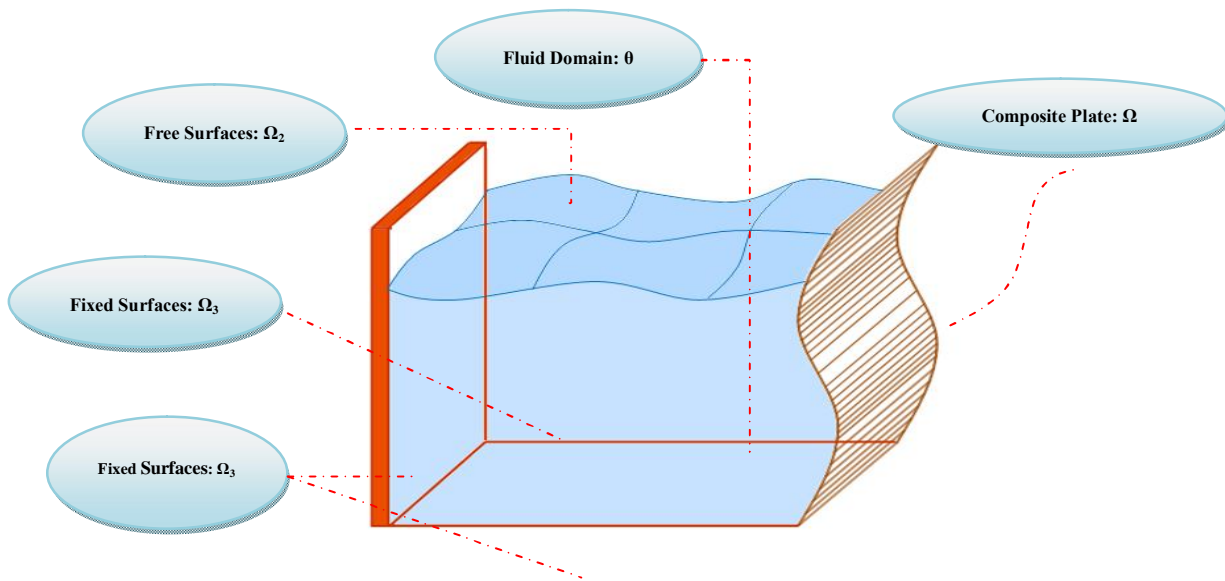


Fig. 1. Schematic view of the fluid-structure problem

2) A free surface with prescribed external pressure where we allow the linearized (gravitational) waves, Ω_2 , see Fig. 1, [17]:

$$\rho_0 \phi_{,tt} + \rho_0 g \phi_{,n} + p_{e,t} = 0 \quad (4)$$

3) Fixed surface with prescribed external pressure, Ω_3 , see Fig.1, [14]:

$$\phi_{,n} = 0 \quad (5)$$

where, $\phi(x, y, z, t)$ is the velocity potential. This means that $\mathbf{v} = \nabla\phi$ and c is the sound speed in the fluid.

b) Structure Domain

The governing equations of a composite plate with external pressure $p(x, y, t)$ can be written as follows [15]:

$$\left. \begin{array}{l} d_{11} w_{,xxxx} + 2(d_{12} + 2d_{66}) w_{,xxyy} + d_{22} w_{,yyyy} + 4d_{16} w_{,xxyy} + 4d_{26} w_{,xyyy} + \rho h w_{,tt} = p \quad \text{in } \Omega \\ w = \partial w / \partial n = 0 \quad \text{in } \Gamma_0 \\ V^{(n)} + \partial M^{(ns)} / \partial s = U_1 \quad \text{in } \Gamma_1 \quad \text{and} \quad M^{(n)} = U_2 \end{array} \right\} \quad (6)$$

$\forall(x, y, t) \in \Omega \times [0, \infty)$; where $w(x, y, t)$ represents the transverse displacement, $p(x, y, 0, t)$ is the external transverse force distribution (hydrodynamic pressure due to fluid loading) on the composite plate, h is the thickness of the composite plate, E_i is the Young's modulus of elasticity (see the proof of Lemma 1.), ν is the poisson's ratio and d_{ij} 's are composite coefficients of elasticity, [15]. It should be noted that Ω is a bounded simple region and $\mathbf{n} = (n_1, n_2)$ is the unit outward normal vector to the boundaries of the composite plate. M_{11} , M_{12} , M_{22} and V_1 , V_2 are defined in the proof of Lemma 1.

3. STABILIZATION OF COMPOSITE PLATE UNDER HEAVY FLUID LOADING

In this section, we consider the stabilization problem of the vibration of a composite plate without any boundary attachment. For this purpose, first, the following definitions will be employed.

The inner product on the space $\mathbf{H} = H^1_{\Omega_3}(\Theta) \times L^2(\Theta) \times H^2_{\Gamma_0}(\Omega) \times L^2(\Omega)$ will be presented as

$$\begin{aligned} \langle X, Y \rangle_{\mathbf{H}} &= \int_{\Theta} \left[\frac{\rho_0}{2c^2} \tau_1 \tau_2 + \frac{\rho_0}{2} \Sigma(\kappa_1, \kappa_2) \right] d\theta \\ &+ \int_{\Omega} [(\rho_0/2g)\tau_1 \tau_2 + (\rho h/2)\zeta_1 \zeta_2 + \Psi(\eta_1, \eta_2)] d\Omega \end{aligned} \tag{7}$$

where $X, Y \in \mathbf{H}$, $X = (\kappa_1, \tau_1, \eta_1, \zeta_1), Y = (\kappa_2, \tau_2, \eta_2, \zeta_2)$, $H^2_{\Omega_3}(\Theta) = \{\kappa_1 : \kappa_1 \in H^2(\Theta) : \partial \kappa_1 / \partial n = 0|_{\Omega_3}\}$ and $H^2_{\Gamma_0}(\Omega) = \{\xi_1 : \xi_1 \in H^2(\Omega) : \xi_1 = 0|_{\Gamma_0}, \partial \xi_1 / \partial n|_{\Gamma_0} = 0\}$; also the following relations hold

$$\begin{cases} \Pi(\kappa_1, \kappa_2) = \kappa_{1,x} \kappa_{2,x} + \kappa_{1,y} \kappa_{2,y} + \kappa_{1,z} \kappa_{2,z} \\ \Lambda(\eta_1, \eta_2) = (1/2)(d_{11}\eta_{1,xx} \eta_{2,xx} + d_{22}\eta_{1,yy} \eta_{2,yy} + d_{12}(\eta_{1,xx} \eta_{2,yy} + \eta_{1,yy} \eta_{2,xx})) \end{cases} \tag{8}$$

where L^2 is the standard Lebesgue functional space and H^n is the standard Hilbert space [16].

It should be noted that $\Lambda(\eta, \eta)$ and $\Pi(\kappa, \kappa)$ take the roles of the strain energy of the composite plate and fluid respectively and therefore must be nonnegative.

The composite plate governing equations and related boundary conditions are as follows, see [17].

$$\left\{ \begin{aligned} &d_{11}w_{,xxxx} + 2(d_{12} + 2d_{66})w_{,xyxy} + d_{22}w_{,yyyy} + 4d_{16}w_{,xxyy} + 4d_{26}w_{,xyyy} + \rho h w_{,tt} = p \quad \text{in } \Omega \\ &w = \partial w / \partial n = 0 \quad \text{in } \Gamma_0 \\ &V^{(n)} + \partial M^{(ns)} / \partial s = U_1 \quad \text{in } \Gamma_1 \quad \text{and} \quad M^{(n)} = U_2 \end{aligned} \right\} \tag{9}$$

For this problem, our main intention is to show that the system (9) under boundary feedback $U_1 = -w_{,t}$ and $U_2 = \partial(w_{,t}) / \partial n$ ($\partial w / \partial n$ is the normal derivative of w) is well-posed and asymptotically stable. Note that $\partial \Omega = \Gamma = \Gamma_1 \cup \Gamma_2$ and

$$M^{(n)} = M_{11}n_1^2 + M_{22}n_2^2 - 2M_{12}n_1n_2 \tag{10}$$

$$M^{(ns)} = (M_{11} - M_{22})n_1n_2 + M_{12}(n_1^2 - n_2^2) \tag{11}$$

$$V_1 = M_{11,x} + M_{12,y} \tag{12}$$

$$V_2 = M_{12,x} + M_{22,y} \tag{13}$$

$$V^{(n)} = V_1n_1 + V_2n_2 \tag{14}$$

where $\vec{n} = (n_1, n_2)$ is the unit outward vector normal to the boundary. V_1, V_2 stand for transversal forces which lay in the planes being perpendicular to unit vectors in x and y directions. $V^{(n)}, M^{(n)}$ are, respectively, transverse force and bending moment which lay being perpendicular to the normal direction. For definitions of the remaining parameters, see the proof of Lemma 1.

To analyze the system using the notion of the linear operators, we utilize the following notation

$$AX = \begin{pmatrix} \tau_1 \\ c^2 \Delta \kappa_1 \\ \zeta_1 \\ \frac{-1}{\rho h} \Sigma(\eta_1) + p \end{pmatrix} \tag{15}$$

where,

$$\Sigma(\xi) = d_{11}\xi_{,xxxx} + 2(d_{12} + 2d_{66})\xi_{,xyxy} + d_{22}\xi_{,yyyy} + 4d_{16}\xi_{,xxyy} + 4d_{26}\xi_{,xyyy}$$

The state space representation of the system of Eq. (9) is:

$$\left. \begin{aligned} \dot{\Xi} &= A\Xi \\ w &= 0, \partial w / \partial n = 0 \quad \text{in } \Gamma_0 \\ V^{(n)} + M^{(ns)}_{,s} &= -w_{,t}, \quad M^{(n)} = \partial(w_{,t}) / \partial n \quad \text{in } \Gamma_1 \\ \rho_0 \phi_{,t} &= -p \quad \text{in } \Omega \\ \rho_0 \phi_{,tt} + \rho_0 g \phi_{,n} &= 0 \quad \text{in } \Omega_2 \\ \phi_{,n} &= 0 \quad \text{in } \Omega_3 \\ \Xi(0) &= \Xi_0 \end{aligned} \right\} \quad (16)$$

where $\Xi = (\xi_1, \xi_2, \xi_3, \xi_4)$, $\phi = \xi_1$, $\phi_{,t} = \xi_2$, $w = \xi_3$ and $w_{,t} = \xi_4$.

At first, it will be shown that the operator A with the following domain is a dissipative operator.

$$\begin{aligned} D(A) = \{ & (\xi_1, \xi_2, \xi_3, \xi_4) \mid \xi_1 \in H^2(\Theta) \cap H^1_{\Omega_3}(\Theta), \xi_2 \in H^1_{\Omega_3}(\Theta), \\ & \xi_3 \in H^2_{\Gamma_0}(\Omega) \cap H^4(\Omega), \xi_4 \in H^2_{\Gamma_0}(\Omega) \text{ such that } \rho_0 \xi_2|_{\Omega} = -p \} \end{aligned} \quad (17)$$

where $H^4_{\Gamma_0}(\Omega) = \{ \xi_3 : \xi_3 \in H^4(\Omega) : \xi_3 = 0|_{\Gamma_0}, \partial \xi_3 / \partial n|_{\Gamma_0} = 0 \}$ and $H^2_{\Omega_3}(\Theta) = \{ \xi_1 : \xi_1 \in H^2(\Theta), \xi_1 = 0|_{\Omega_3} \}$.

Lemma 1. A is a dissipative operator.

Proof. We start from the fact that the total mechanical energy of the systems is equal to the following inner product $E(t) = \langle \Xi, \Xi \rangle$, therefore

$$\dot{E}(t) = 2 \langle \Xi, \dot{\Xi} \rangle = 2 \langle \Xi, A\Xi \rangle \quad (18)$$

For the operator A , with definition (15), one can have

For the rest of the proof, we define some parameters, [15],

$$M_{11} = -d_{11}w_{,xx} - d_{12}w_{,yy} - 2d_{16}w_{,xy} \quad (19)$$

$$M_{22} = -d_{12}w_{,xx} - d_{22}w_{,yy} - 2d_{26}w_{,xy} \quad (20)$$

$$M_{12} = -d_{16}w_{,xx} - d_{26}w_{,yy} - 2d_{66}w_{,xy} \quad (21)$$

$$\kappa_{11} = -w_{,xx}, \quad \kappa_{22} = -w_{,yy}, \quad \kappa_{12} = -2w_{,xy} \quad (22)$$

$$V_1 = M_{11,x} + M_{12,y} \quad (23)$$

$$V_2 = M_{12,x} + M_{22,y} \quad (24)$$

We notice that the governing equation of motion can be rewritten in the following form, [15],

$$M_{11,xx} + 2M_{12,xy} + M_{22,yy} = \rho h w_{,tt} \quad (25)$$

The energy functional takes the following form

$$\begin{aligned} E(t) = & \frac{1}{2} \int_{\Omega} [M_{11} \kappa_{11} + M_{22} \kappa_{22} + M_{12} \kappa_{12} + \rho h w_{,t}^2] d\Omega \\ & + \int_{\Theta} \left[\frac{\rho_0}{2c^2} \phi_{,t}^2 + \frac{\rho_0}{2} |\nabla \phi|^2 \right] d\Theta + \int_{\Omega_2} \left[\frac{\rho_0}{2g} \phi_{,t}^2 \right] d\Omega \end{aligned} \quad (26)$$

Therefore, time derivative of $E(t)$ will be

$$\begin{aligned} \dot{E}(t) = & \frac{1}{2} \int_{\Omega} [\dot{M}_{11} \kappa_{11} + M_{11} \dot{\kappa}_{11} + \dot{M}_{22} \kappa_{22} + M_{22} \dot{\kappa}_{22} + \dot{M}_{12} \kappa_{12} + M_{12} \dot{\kappa}_{12} + 2\rho h w_{,t} w_{,tt}] d\Omega \\ & + \int_{\Omega_2} \left[\frac{\rho_0}{g} \phi_{,t} \phi_{,tt} \right] d\Omega + \int_{\Theta} \left[\frac{\rho_0}{c^2} \phi_{,t} \phi_{,tt} + \rho_0 (\phi_{,tx} \phi_{,x} + \phi_{,ty} \phi_{,y}) \right] d\Theta \end{aligned} \quad (27)$$

and therefore

$$\begin{aligned} 2\dot{E}(t) = & \int_{\Omega} [\dot{M}_{11} \kappa_{11} + \dot{M}_{22} \kappa_{22} + \dot{M}_{12} \kappa_{12} + (M_{11,xx} + 2M_{12,xy} + M_{22,yy}) w_{,t}] d\Omega \\ & + \int_{\Omega} [\dot{M}_{11} \kappa_{11} + \dot{M}_{22} \kappa_{22} + \dot{M}_{12} \kappa_{12} + (M_{11,xx} + 2M_{12,xy} + M_{22,yy}) w_{,t}] d\Omega \\ & \int_{\Omega_2} \left(\frac{\rho_0}{g} \phi_{,t} \phi_{,tt} \right) d\Omega + \rho_0 \int_{\Theta} \left[\frac{\partial}{\partial x} (\phi_{,t} \phi_{,x}) + \frac{\partial}{\partial y} (\phi_{,t} \phi_{,y}) \right] d\Theta \end{aligned} \quad (28)$$

Employing the relations for the resultant moments in directions x and y , see Eqs. (18) to (24),

$$\begin{aligned} 2\dot{E}(t) = & \int_{\Omega} [(M_{11,xx} w_{,t} - M_{11} w_{,xxt}) + (M_{22,yy} w_{,t} - M_{22} w_{,yyt}) + 2(M_{12,xy} w_{,t} - M_{12} w_{,xyt})] d\Omega \\ & + \int_{\Omega} D[w_{,xx} w_{,xxt} + \nu w_{,yyt} w_{,xx}] d\Omega + \int_{\Omega} D[\nu w_{,yy} w_{,xxt} + w_{,yyt} w_{,yy}] d\Omega \\ & + \int_{\Omega} 2D(1-\nu) w_{,xyt} w_{,xy} d\Omega - \int_{\Omega} D[w_{,xxx} w_{,t} + \nu w_{,xxy} w_{,t}] d\Omega \\ & - \int_{\Omega} D[\nu w_{,xyy} w_{,t} + w_{,yyy} w_{,t}] d\Omega - 2 \int_{\Omega} D(1-\nu) w_{,xxy} w_{,t} d\Omega \\ & + \int_{\Omega} p w_{,t} d\Omega + \int_{\Omega_2} \frac{\rho_0}{g} \phi_{,t} \phi_{,tt} d\Omega + \rho_0 \int_{\Omega} \phi_{,t} \phi_{,n} d\Omega + \rho_0 \int_{\Omega_2} \phi_{,t} \phi_{,n} d\Omega + \rho_0 \int_{\Omega_3} \phi_{,t} \phi_{,n} d\Omega \end{aligned} \quad (29)$$

Rearranging the terms and using the boundary conditions for the fluid yield

$$\begin{aligned} 2\dot{E}(t) = & 2 \int_{\Omega} [(M_{11,x} w_{,t} + M_{12,y} w_{,t} - M_{11} w_{,tx} - M_{12} w_{,yt})_{,x}] d\Omega \\ & \int_{\Omega} [(M_{22,y} w_{,t} + M_{12,x} w_{,t} - M_{22} w_{,ty} - M_{12} w_{,xt})_{,y}] d\Omega \\ & + \int_{\Omega} p w_{,t} d\Omega + \int_{\Omega_2} \frac{\rho_0}{g} \phi_{,t} \phi_{,tt} d\Omega + \rho_0 \int_{\Omega} \phi_{,t} \phi_{,n} d\Omega + \rho_0 \int_{\Omega_2} \phi_{,t} \phi_{,n} d\Omega \end{aligned} \quad (30)$$

Applying Green's Lemma and also boundary conditions of the fluid yield

$$\begin{aligned} \dot{E}(t) = & \int_{\Gamma} (M_{11,x} w_{,t} + M_{12,y} w_{,t} - M_{11} w_{,tx} - M_{12} w_{,yt}) n_1 d\Gamma \\ & + \int_{\Gamma} (M_{22,y} w_{,t} + M_{12,x} w_{,t} - M_{22} w_{,ty} - M_{12} w_{,xt}) n_2 d\Gamma \\ & + \int_{\Omega} p w_{,t} d\Omega - \rho_0 \int_{\Omega_2} \phi_{,t} \phi_{,n} d\Omega - \int_{\Omega} p w_{,t} d\Omega + \rho_0 \int_{\Omega_2} \phi_{,t} \phi_{,n} d\Omega \end{aligned} \quad (31)$$

Grouping the terms and noting that

$$\frac{\partial \Delta}{\partial x} = n_1 \frac{\partial \Delta}{\partial n} - n_2 \frac{\partial \Delta}{\partial s} \quad (32)$$

$$\frac{\partial \Delta}{\partial y} = n_1 \frac{\partial \Delta}{\partial s} + n_2 \frac{\partial \Delta}{\partial n} \quad (33)$$

yield the following result

$$\dot{E}(t) = \int_{\Gamma} [(V^{(n)} + \partial M^{(ns)}/\partial s) w_{,t} - M^{(n)}(w_{,t})_{,n}] d\Gamma \quad (34)$$

By applying the boundary controllers mentioned in Eq. (16), and using the related boundary conditions, the following result is attained:

$$\dot{E}(t) = -\|\xi_4\|_{L^2(\Gamma_1)}^2 - \|\partial \xi_4 / \partial n\|_{L^2(\Gamma_1)}^2 \leq 0 \quad (35)$$

With the above premise, the proof will be complete.

Lemma 2. The resolvent $(\alpha I - A)^{-1}$ exists and is compact ($\forall \alpha > 0$).

For this purpose, we utilize the following relation

$$(\alpha I - A)X = X_0, \quad X_0 \in \mathbf{H} \quad (36)$$

It can be seen that

$$\langle (\alpha I - A)X, X \rangle_{\mathbf{H}} = \alpha \|X\|_{\mathbf{H}}^2 + \|\xi_4\|_{L^2(\Gamma_1)}^2 + \|\partial \xi_4 / \partial n\|_{L^2(\Gamma_1)}^2 \geq \alpha \|X\|_{\mathbf{H}}^2 \quad (37)$$

where $\|X\|_{\mathbf{H}}^2 = \langle X, X \rangle$.

Using Lax–Milgram Lemma, one can easily prove that the above equation has a unique weak solution, see [16, 18 and 19]. In particular one can infer that:

$$R(\alpha I - A) = H^2(\Theta) \times H^1(\Theta) \times H^4(\Omega) \times H^2(\Omega), \text{ where } \alpha > 0.$$

On the other hand, it is clear that $D(A)$ is dense in $H^2(\Theta) \times L^2(\Theta) \times H^4(\Omega) \times L^2(\Omega)$, hence, according to Lumer-Phillips theorem, it is proved that A generates a C_0 -semigroup of contractions (see [20]). Finally, one can obtain the following result

$$\|X_0\|_{\mathbf{H}} \geq \alpha \|X\|_{\mathbf{H}} \quad (38)$$

Using Sobolev embedding theorem (Rellich-Kondrachov compact embedding theorem [19]), since $(\alpha I - A)^{-1}V$ is compactly embedded in $L^2(\Theta) \times L^2(\Theta) \times L^2(\Omega) \times L^2(\Omega)$, the compactness of the above-mentioned resolvent is evident.

Theorem 1. Let in the system (16), the initial condition Ξ_0 belong to $D(A)$. Then, the system of Eq. (16) is well-posed.

Proof. Based on Lemma 2, it is evident that the system (16) is well-posed [20]. In addition, its strong solution has the following regularity; see [20 and 21].

$$\begin{aligned}\phi(t) &\in C^0([0, t], H^2(\Theta) \cap H_{\Omega_3}^1(\Theta)) \cap C^1([0, t], H_{\Omega_3}^1(\Theta)) \cap C^2([0, t], L^2(\Theta)) \\ w(t) &\in C^0([0, t], H^4(\Omega) \cap H_{\Gamma_0}^2(\Omega)) \cap C^1([0, t], H_{\Gamma_0}^2(\Omega)) \cap C^2([0, t], L^2(\Theta))\end{aligned}$$

Now, we turn our attention to the proof of the asymptotic stability of the closed loop system.

Theorem 2. Using the boundary feedback control laws (40), the states of the system, Ξ , will eventually tend toward zero.

$$U_1 = -\xi_4 \quad \text{and} \quad U_2 = \partial \xi_4 / \partial n \quad (39)$$

Proof. The mechanical energy of the system as discussed previously, is

$$E(t) = \langle \Xi, \Xi \rangle \quad (40)$$

By performing some algebraic operations and using Green's Lemma, the following can be obtained (see the proof of Lemma 1.):

$$\dot{E}(t) = -\|\xi_4\|_{L^2(\Gamma_1)}^2 - \|\partial \xi_4 / \partial n\|_{L^2(\Gamma_1)}^2 \leq 0 \quad (41)$$

At this step, because of the compactness of resolvent $(\alpha I - A)^{-1}$, one can use LaSalle's invariant set theorem and therefore, it is sufficient to show that the following system has the trivial solution as its unique solution.

$$\left. \begin{aligned} \dot{\Xi} &= A \Xi \quad \text{in} \quad \Omega \\ \xi_4 &= \partial \xi_4 / \partial n = 0 \quad \text{and} \quad M^{(n)} = V^{(n)} = 0 \quad \text{in} \quad \Gamma_1 \\ \xi_3 &= \partial \xi_3 / \partial n = 0 \quad \text{in} \quad \Gamma_0 \\ \rho_0 \phi_{,t} &= -p \quad \text{in} \quad \Omega \\ \rho_0 \phi_{,tt} + \rho_0 g \phi_{,n} &= 0 \quad \text{in} \quad \Omega_2 \\ \phi &= 0 \quad \text{in} \quad \Omega_3 \\ \Xi(0) &= \Xi_0 \end{aligned} \right\} \quad (42)$$

Using the Holmgren uniqueness theorem, [22], one can easily show that the above system of equations admits only trivial solution. Then, by regarding the LaSalle's invariant set theorem,

$$\lim_{t \rightarrow \infty} E(t) = 0 \quad (43)$$

which yields the desired stability.

4. CONCLUSION

A boundary control law was designed to stabilize the vibration of composite plates in contact with a fluid. Asymptotic stability was proved using semigroup techniques and Lyapunov method. It was shown that the mechanical energy of the systems would converge asymptotically toward zero. Since the control laws

consisted only of the feedback from the shear force and bending moment at the boundary of composite plate, measurement and costs were minimized.

Also, the proposed method avoids installation of distributed actuators/sensors which meant observation of vibration data along the composite plate or in the interior of the fluid was not required and the asymptotical stability of the fluid was accomplished without using any actuation in the fluid domain or its boundary.

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