"Research Note"

VIBRATION CHARACTERISTICS OF CONTINUOUSLY VARIABLE TRANSMISSION PUSH BELTS

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Abstract—A CVT push-belt, composed of 12 layers of bands and a number of segments, is modeled for vibration analysis. Predefined compression and tension loads are applied to segments and bands respectively. Continuous and discrete beams composed of segments are used to investigate contact properties between segments. Three models of band contacts have been established based on complexity and various modeling approaches in ABAQUS. Contacts between bands are modeled by a frictionless contact between shell elements tied together in the first model and in the second model. To improve the accuracy, a composite shell with special interface layers is utilized as third model. By using the segment models, it has been concluded that a group of segments can be considered as a continuous beam at high compression loads and as individual rigid masses in low compression loads. The results of composite shell model show better prediction of the vibration properties of bands pack model. Separate cases of loading are considered for tension and compression spans.

Keywords—CVT, push-belt, vibration, finite element, contact modeling

1. INTRODUCTION

A push-belt CVT consists of two sets of pulleys and the belt which runs inside them. The speed-ratio varies by changing the radial belt position towards the pulley center. The belt consists of two sets of bands and a large number of segments. A set of bands consists of 12 layers which fit without play (Fig. 1). The transmitted torque results in the compression force between the elements as well as the ring tension.

Dynamic modeling of CVT belts have been dealt with by researchers in areas of load analysis and vibration modeling. Kuwabara et al [1] proposed a theoretical and experimental modeling for obtaining load distribution between bands. This model was composed of a multilayer belt without segments. They concluded that for having a better distribution of forces between layers, the friction coefficient between layers should be slightly higher than that between inner layer and segments. In other work for push-belt by using a numerical model, forces in CVT belts were obtained [2]. Thrust ratio is shown to become longer with an increase in coefficient of friction. Sirvastava and Haque [3] studied the transient dynamics of push-belt by investigating the effects of bands sliding on dynamic characteristics of belts.

Lebrecht, Pfeiffer and Ulbrich [4] investigated the self-induced vibration in a CVT push-belt. They indicated that a friction characteristic, depending on the relative velocity between the elements and the pulleys can cause self-induced vibrations of the belt. Among all the works in this subject, Lebrecht’s paper is the only case which deals with vibration analysis of push-belts and presented a simple analytical model for deriving natural frequencies of a push-belt.

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The majority of works are related to joints with small dimensions in comparison with the entire part dimensions. Ahmadian and Jalali [5] used linear and non-linear springs and dampers for modeling bolt elements between two beams. The parameters of the joint interface model were identified by minimizing the difference between the measured responses and the model predictions. Spot-welded joints in thin layers have also been an issue of interest [6]. The results showed that the detailed or the single beam models were relatively inaccurate. Ahmadian et al [7] studied the large surface-surface contacts in a typical structure by using a thin layer of auxiliary material at the contact region. They showed that the model is able to present torsional and longitudinal mode shapes sufficiently. The frictionless contact for a nonsymmetrical two dimensional elasticity problem of an elastic layer which is supported by two elastic quarter planes has been carried out by Aksogan et al [8].

In the current work, an FE model of push-belt is proposed to obtain vibrational characteristics of the belt in designing stage. The contacts between bands and between segments as well as outer bands and segments are considered in modal analysis.

2. MODELING

The part of the belt over the pulleys is intensively pressed, and as a result, it is unable to vibrate at low frequencies. The span of the belt which connects the two pulleys can, however, fluctuate in much lower frequencies. Hence, the critical natural frequencies happen in the spans. For the purpose of modeling, the belt is considered to be hinged at the two ends of contact points with the pulleys (Fig. 2). Appropriate load and boundary conditions should be applied to this model in order to consider the effects of bending, velocity, as well as tension in bands and compression in segments. In the modeling procedure, the effects of belt speed are small (the main reason is the direction of centripetal acceleration on the bends that is normal to the belt direction) and can be neglected [4]; this has not been taken into account in this paper. Owing to the relatively small effects and for the sake of simplicity, the boundary moments are not applied at the beam ends.

![Image](image_url)

Fig. 2. A schematic of the belt and simplified model

**a) Theoretical model**

In this section, a mathematical modeling of a hinged beam used by Lebrecht et al [4] is presented and will be employed for comparison purposes. The free diagram of a section of a belt is shown in Fig. 3.
By using the Newton’s second law one can write:

\[ m \frac{\partial^2 u}{\partial t^2} dx = -(V + \frac{\partial V}{\partial x} dx) + V + T(\theta + \frac{\partial \theta}{\partial x} dx) - T\theta \]  

(1)

Where \( m \) is mass per length, \( \theta = \frac{\partial u}{\partial x} \), \( V \) is shear force, \( u = u(x,t) \) deflection and \( T \) is the tension in beam, which is negative for comparison forces. Using equilibrium equations and solving the differential equation will lead to the natural frequencies:

\[ \omega_n^2 = \left( \frac{n \pi}{l} \right)^2 \frac{T}{m} \left( \frac{n \pi}{l} \right)^2 \frac{EI}{m} \]  

(2)
in which \( E \) is modulus of elasticity, \( I \) is the second moment of the section area, \( l \) is length of the beam and \( n \) refers to natural frequency number.

**Fig. 3. Free body diagram of a section of the belt**

\[ b) Finite element modeling and mesh generation \]

A segment model and a band layer model are demonstrated in this section. The complete model is composed of a set of 12-layer bands and 100 segments considering band-band, segment-segment, and band-segment contacts. Due to symmetrical properties of the belt, only half of the belt is modeled and segments are sectioned from the symmetric plane. Due to the small thickness of bands, shell elements are the best choice for the FE modeling of bands. In order to increase the accuracy, all shell elements used in the model are rectangular quadratic cases (Fig. 4a). For segment modeling improper brick elements may cause a large inaccuracy and thus hexagonal elements are used in FE model of segments to avoid element distortion. The meshed model and element properties of a segment are shown in Fig. 4b.

**Fig. 4. Meshed models of (a) band and (b) segment**

**3. CONTACT BETWEEN SEGMENTS**

In order to investigate the transverse contacts among the segments, two cases are considered. The first case is a split beam composed of an assembly of segments resembling the real case. The second model is a
continuous beam with similar mass properties with the first model. In fact, in the first model each two adjacent segments are in contact with each other and FE model considers contact elements, whereas in the second model segments are bonded together to make a single continuous beam.

The natural frequencies of the two models are depicted in Fig. 5. It shows that if the compression force increases, these two results become closer and the difference becomes less than 1%. Moreover, when the compression force is low, the natural frequency of the split beam is near to zero whereas continuous model shows non-zero natural frequencies in this force. However, in natural frequencies, a specific point near to zero can be considered as the critical load, as it is shown in Fig. 5b. Since low compression force segments show rigid modes, it is claimed that in low compression force, modeling assembly of segments as separate masses without elasticity seems reasonable.

Fig. 5. Split and continuous beam frequencies, (a) & (b) First, (c) Second, and (d) Third natural frequencies

4. CONTACT BETWEEN BANDS

In a frictionless model, every band slides easily on its neighboring band. The mode shapes obtained for this model were similar to those of a continuous beam. Since every band is free to slide, the bands assembly differs from multilayer bands which are tied together. The natural frequencies of the frictionless model are very similar to those of a single band. A tied model is the one in which all of the bands are tied firmly together making a single band of larger thickness. In this model, layers are not allowed to slide relative to their neighbors; therefore, it is quite similar to a thick layer with the thickness of the set.

In order to account for the contact forces between every two bands, the thin layer method which was previously employed by Ahmadian [7] is adopted in the third model. A thin layer of a special material with properties found from experiments is used to model the contact elements. In this composite multilayer beam, bands can move with respect to their neighbors. In fact, the displacements take place in the auxiliary layers.

For the contact layer or pseudo-layer model, each contact layer has given a thickness of 0.02 mm and its mechanical properties are taken E=9.75E8 MPa and G=5.42E8 MPa. Fig. 6 compares the first three
natural frequencies of the three models with increasing tension. As it can be seen, the lowest values belong to frictionless model. The percentages of differences in the first three natural frequencies of the models have also been compared in Fig. 7. In this figure, the contact layer model is considered as the base model and the differences between each model results with respect to those of the base model are evaluated; consequently, the horizontal axis shows pseudo-layer model properties.

Fig. 6. First three natural frequencies of three longitudinal contact models, (a) First, (b) Second, and (c) Third Natural Frequencies

Fig. 7. Percentage of differences of first three natural frequencies relative to contact layer model, (a) First, (b) Second, and (c) Third Natural Frequencies

In higher frequency numbers, differences become more recognizable and the differences are high in low tension loads. By increasing the load, differences decline. It is also revealed that tied layers model results are close to those of contact layer model.

The contact or pseudo layer thickness is varied from the initial value of 0.02 mm to values 0.01, 0.04 and 0.06 mm. The results obtained for these three cases with the same mechanical properties are compared with the basic case. The percentages of differences of three cases with respect to the base case are shown in Fig. 8. Maximum difference is seen to be about 9% in very low forces and the difference vanishes at high values of forces.

Fig. 8. Percentage of differences of first natural frequencies for different thickness of contact layer, (a) First, (b) Second, (c) Third Natural Frequencies
5. CONCLUSION

A Finite Element modeling of a CVT push-belt was developed including its bands and segments. By comparing a split and continuous beam model, it was shown that at high compression force, segments can be accurately modeled as a continuous beam while in low compression loads, segments in the belt act like individual masses. An auxiliary layer approach was utilized for modeling contact effects between the bands. Two other extreme models were also developed for comparison reasons, one with no friction between layers and a tied layer model. Despite the differences between three band models in low tension load, they showed closer results in high tension loads. The effect of pseudo-layer properties on the accuracy of the contact modeling was also studied. It was found that shear modulus of pseudo-layer was more influential than its elasticity or thickness.

REFERENCES