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COMBINED RADIATIVE -CONDUCTIVE HEAT TRANSFER IN TWO-DIMENSIONAL COMPLEX GEOMETRIES WITH VARIABLE THERMAL CONDUCTIVITY*

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Abstract– This article deals with a simple Cartesian practical method named blocked-off procedure to study the steady state combined conductive-radiative heat transfer in two-dimensional irregular geometries. Using this technique, both straight and curvilinear boundaries can be treated. The finite-volume method is employed to solve the energy equation and the discrete ordinates method (DOM) is used to solve the radiative transfer equation (RTE) to obtain the temperature and radiative-conductive heat flux distributions inside the participating medium. The walls of enclosures are opaque, diffuse and gray with specified temperatures. The medium was considered to be absorbing-emitting and isotropic scattering with variable thermal conductivity. In the case of constant thermal conductivity, results have been compared with those reported in the literature. The major part of this work is to investigate the effects of variable thermal conductivity on the thermal characteristics of radiative-conductive systems.

Keywords– Complex geometries, radiation, conduction, blocked-off method, DOM

1. INTRODUCTION

Combined radiation and conduction heat transfer in an absorbing, emitting and scattering medium is important in many engineering applications such as heat transfer process in glasses, thermal insulation materials and optical measurement of flame, etc. Most of the previous studies dealt with combined conduction-radiation heat transfer considering constant thermal conductivity for simple geometries. For numerical solution of the radiative transfer equation (RTE), different numerical schemes including the finite volume method (FVM), discrete ordinates method (DOM), zone method, Monte Carlo method, flux method, discrete transfer method and P_N method have been used. None of these methods is superior to the others in all aspects and every method has its own relative benefits and disadvantages. About the effects of variable thermal conductivity in a combined radiative-conductive thermal system limited literature is available [1, 2]. For this reason, no work has been reported so far that deals with the thermal analysis of radiative-conductive heat transfer in a 2-D complex geometry area by considering variable thermal conductivity.

The method of discrete ordinates used in the present work is an accurate simplified method to solve radiative transfer problems. This technique was originally formulated by Chandrasekar in 1950 [3], and has been deeply studied by Carlson and Lathrop in the 60-70's [4] and by Fiveland and Truelove in the 80's [5, 6].

Up to now, the combined radiation and conduction heat transfer in participating medium has been solved by many investigators. For example, Yuen and Wong [7] investigated the influence of the anisotropic scattering on combined heat transfer in one-dimensional planar geometry. Ismail and Salinas [8] used the DOM with a multi-dimensional spatial scheme for the radiative part and used the finite-

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volume method for solving the energy equation. Control-Volume Finite Element Method (CVFEM) has been used to analyze combined conduction-radiation problem in two-dimensional cavities by Rousse et al. [9]. The DOM was used for analyzing combined convection radiation heat transfer in separated duct flows by the second author [10]. The product discrete ordinates method found its application in the work of Kim et al. [11] for solving combined conduction-radiation problem in rectangular enclosures. Talukdar et al. [12] used the collapse dimension method for the solution of combined mode of heat transfer. Lee and Viskanta [13] compared the solutions of the combined conductive-radiative heat transfer in the two-dimensional semitransparent media using the finite-volume method for the energy equation coupled with the DOM and diffusion approximations for the RTE.

In order to avoid the complexity of treating the non-orthogonal grids for the irregular geometry, it is suitable to formulate a procedure to model the irregular geometries using the Cartesian coordinates. In the computational fluid dynamics (CFD) problems, the concept of blocked-off region was applied previously by Patankar [13]. In participating media, Chai et al. [14, 15] discussed different possibilities of solving radiative transfer problems in irregular structures using the discrete ordinates method and the finite volume method. Talukdar [16] analyzed the two-dimensional irregular geometries with the concept of blocked-off region using the Discrete Transfer Method (DTM). He found that the method of block-off can be recommended as a good alternative to solving problems with irregular geometries.

In all of the above researches, the thermal conductivity of the radiating medium was assumed to be constant. It is evident that this assumption may lead to inaccurate results in a media with large temperature gradients. For this purpose, the present study deals with numerical analysis of radiative-conductive heat transfer in irregular shape enclosures in which the variation of thermal conductivity of the participating media with temperature is taken into account. In this paper, a general formulation of the DOM and the FVM to analyze conduction–radiation heat transfer with variable thermal conductivity is presented. To validate the numerical findings, some representative results are compared with those available in the literature for constant thermal conductivity. Moreover, effects of variable thermal conductivity on thermal behavior of participating media inside a T shape furnace with heat source are carried out.

2. PROBLEM FORMULATION

The energy equation for coupled radiation–conduction heat transfer of an absorbing, emitting and scattering media under steady state condition with variable thermal conductivity and heat generation within a radiating medium is as in Siegel and Howell [17]:

$$\nabla.(\mathrm{k}\nabla \mathrm{T}) - \nabla.\mathrm{q_r} + \dot{\mathrm{Q}}''' = 0 \tag{1.a}$$

In this study, the above equation is estimated by the following one, but by considering variable thermal conductivity as a function of temperature:

$$\mathrm{k}\nabla^2 \mathrm{T} - \nabla.\mathrm{q_r} + \dot{\mathrm{Q}}''' = 0 \tag{1.b}$$

The variation of thermal conductivity with temperature is taken as:

$$\mathrm{k(T)} = \mathrm{k_0} + \alpha' \,(\mathrm{T} - \mathrm{T_{ref}}) \tag{2}$$

Where k_0 is the reference thermal conductivity and α' is the variable thermal conductivity parameter.

a) Radiative problem

In Eq. (1.a), $\nabla.q_r$, is the divergence of the radiative heat flux given by:

$$\nabla.\mathrm{q_r} = \kappa[4\pi \mathrm{I_b}(\mathrm{T(r)}) - \mathrm{G(r)}] \tag{3}$$

here,κ and $I_b$are absorption coefficient and black body radiation intensity respectively. The last term is irradiation that can be computed as follows:

$$G(r) = \int_{4\pi} I(r, \vec{s}) d\Omega \tag{4}$$

To obtain $\nabla . q_r$, it is necessary to solve the radiative transport equation. This equation for an absorbing, emitting and scattering gray medium with isotropic scattering can be written as [18]:

$$(\vec{s}.\nabla)I(r, \vec{s}) = -\beta I(r, \vec{s}) + \kappa I_b(T(r)) + \frac{\sigma_s}{4\pi} G(r) \tag{5}$$

where, I, β and σ_s are radiation intensity, extinction coefficient and scattering coefficient, respectively.

For diffusely reflecting surfaces, the radiative boundary condition is computed by:

$$I(r_w, \vec{s}) = \varepsilon_w I_b(T(r_w)) + \frac{\rho_w}{\pi} \int_{\vec{n}_w . \vec{s}^{in} < 0} I(r_w, \vec{s}^{in}) |\vec{n}_w . \vec{s}| d\Omega^{in} \tag{6}$$

Where r_w, $\vec{n}_w$, ρ_w and ε_w and are the position denoting the boundary surface, outward unit vector normal to the surface, wall reflectively and emissivity of the surface, respectively.

In the discrete ordinates method, Eq. (5) is solved for a set of M different directions $\vec{s}_k$,k=1,2,…,M , and the integrals over direction are replaced by numerical quadratures, that is:

$$\int_{4\pi} f(\vec{s}) d\,\Omega \approx \sum_{k=1}^{M} w_k f(\vec{s}_k) \tag{7}$$

Where w_k is the quadrature ordinates weight associated with direction $\vec{s}_k$. Thus, Eq. (5) is approximated by a set of M equations:

$$(\vec{s}_m.\nabla)I(r, \vec{s}_m) = -\beta I(r, \vec{s}_m) + \kappa I_b(T(r)) + \frac{\sigma_s}{4\pi} G(r)\ , \ m=1,2,\ldots, M \tag{8}$$

subject to the boundary conditions:

$$I(r_w, \vec{s}_m) = \varepsilon_w I_b\big(T(r_w)\big) + \frac{\rho_w}{\pi} \sum_{\vec{n}_w . \vec{s}^{in}{}_m < 0} w_k I(r_w, \vec{s}^{in}{}_m) |\vec{n}_w . \vec{s}_m| \tag{9}$$

Once the intensities have been determined, the value of incident radiation (G) may be found from its definition as follows:

$$G(r) = \int_{4\pi} I(r, \vec{s}) d\Omega = \sum_{k=1}^{M} w_k I(r, \vec{s}_k) \tag{10}$$

In the Cartesian coordinates system, Eq. (8) becomes,

$$\xi_m \frac{\partial I^m}{\partial x} + \eta_m \frac{\partial I^m}{\partial y} + \beta I^m = \beta S^m \tag{11}$$

Where S^m is shorthand for the radiative source function as:

$$S^m = (1 - \omega) I_b(T(r)) + \frac{\omega}{4\pi} G(r) \tag{12}$$

In which $\xi_m$and η_m are the directional cosines of $\vec{s}_m$, and $\omega = (\sigma_s/\beta)$ is the scattering albedo.

By using (DOM) and discretization (RTE) with the help of finite volume method, The volume averaged intensity (I_p^m) for any discrete ordinate (m) of the control volume can be calculated as follows, [18]:

$$I_p^m = \frac{\beta V S_p^m + |\xi_m| A_x I_{x_{in}}^m \gamma_x + |\eta_m| A_y I_{y_{in}}^m \gamma_y}{\beta V + |\xi_m| A_{x_{out}} \gamma_x + |\eta_m| A_{y_{out}} \gamma_y} \tag{13}$$

Where

$$A_x = \left(1-\gamma_x\right)A_{x_{out}} + \gamma_x A_{x_{in}}$$
$$A_y = \left(1-\gamma_y\right)A_{y_{out}} + \gamma_y A_{y_{in}} \quad (14)$$

In which (γ_x) and (γ_y) are spatial differencing weights related to the x and y-directions, respectively and have values between 0.5 and 1.0. We select the step scheme ($\gamma_x = \gamma_y = 1.0$) which is simple, convenient, stable and ensures positive intensities.

Assuming that the boundary conditions are given, the system of equations is closed and defines an interpolation system relating the intensities at the volume to the face values. A two-dimensional Cartesian enclosure has four corners, from each of which $\frac{1}{4}M(\frac{M}{2}+1)$ directions must be traced (covering one quarter of directions), for a total of $M\left(\frac{M}{2}+1\right)$ordinates [18].

3. SOLUTION OF RADIATIVE CONDUCTIVE MODEL

By using a set of reference parameters, the following non-dimensional group can be found in a combined radiation-conduction problem:

$$\Theta = T/T_{ref}, X = \beta x, Y = \beta y, \bar{I} = \frac{I}{\sigma T_{ref}^4}, N_{cr} = \frac{k_0 \beta}{4\sigma T_{ref}^3}, \lambda = \frac{\alpha' \beta}{4\sigma T_{ref}^2}, Q = \frac{q}{\sigma T_{ref}^4}, \dot{Q} = \frac{\dot{Q}'''}{\beta \sigma T_{ref}^4} \quad (15)$$

The energy equation in non-dimensional form is expressed as:

$$(1+\frac{\lambda\,(\Theta-1)}{N})(\frac{\partial^2\Theta}{\partial X^2}+\frac{\partial^2\Theta}{\partial Y^2}) = S_t \quad (16)$$

$$S_t = \frac{(1-\omega)}{N_{cr}}\left(\Theta^4(\mathrm{r}) - \frac{1}{4}\sum_{k=1}^{M} w_k \bar{I}(\mathrm{r},\vec{s}_k)\right) - \frac{\dot{Q}}{4N_{cr}} \quad (17)$$

In these equations, λ is the conductivity-temperature coefficient, N_{cr} is the conduction-radiation parameter, and S_t is total source term.

After determining the intensities and temperature fields, the heat fluxes can be calculated. The dimensionless directional heat fluxes, Q_x and Q_y , including both conduction and radiation are defined as in Kim et al [11]:

$$Q_x = -\frac{4N_{cr}}{\tau}\frac{\partial\Theta}{\partial X} + \sum_{k=1}^{M} w_k \xi_k \bar{I}^k \quad (18-a)$$

$$Q_y = -\frac{4N_{cr}}{\tau}\frac{\partial\Theta}{\partial Y} + \sum_{k=1}^{M} w_k \eta_k \bar{I}^k \quad (18-b)$$

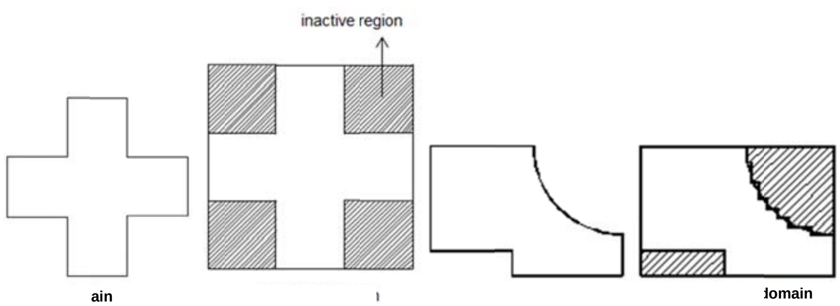
where, the first and second terms on the right-hand side are the dimensionless conductive and radiative heat fluxes.

The energy Eq. (16), which is non-linear is discretized using the finite-volume method as in Patankar [13]. It is worth noting that not only are the governing transport equations coupled, but also their boundary conditions are interlocked. Thus, an iterative solution is needed.

4. THE BLOCKED-OFF METHOD

The combined conductive and radiative heat transfer in the irregular geometries is modeled using the blocked-off method commonly used in the CFD. In the blocked-off method, we draw rectangular nominal domains around the given real domains (simulated domain) and the whole rectangular region is divided into two parts: active and in active or blocked-off regions. The solutions are sought just for active regions.

These regions are un-shaded in Fig. 1a, and b. In these figures, the shaded portion is called the inactive region and solutions are not important. Also, curved boundaries emerge to be a stair step as shown in Fig. 1b. The magnitude of an intensity that enters an inactive region becomes zero and if it enters again to an active region, it is considered as another boundary condition. All the simulated domain is discretized into several control volumes and the control volumes which are inside the active region are intended as one (1), otherwise they are zero (0).This subject is shown in Fig. 2. Moreover, in this method, additional boundary conditions appear because of the extension of irregular geometry into the rectangular shape. So, in the calculations, these boundaries are treated like external boundaries which are sides of the rectangular domain. These boundary conditions are shown as murky color in Fig. 2.



(a) (b)

Fig. 1. Samples of irregular geometries

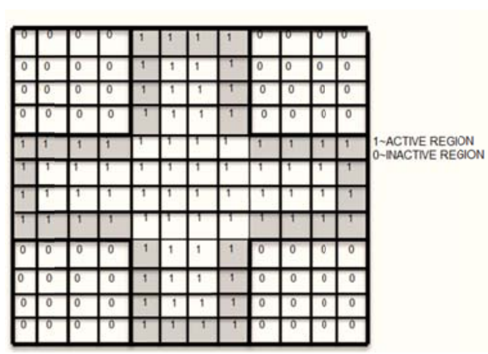


Fig. 2. A domain with blocked-off/inactive region

The procedure of the numerical calculations for temperature distribution can be summarized as follows:

1- Assume zero value for the radiative source term.
2- Assume a temperature distribution over the active region medium.
3- Calculate thermal conductivity by using Eq. (2).
4- Solve the RTE using the DOM method for active control volumes in all directions.
5- Solve the energy equation for the active region to obtain the temperature distribution.
6- Calculate source term by using Eq. (12).
7- Repeat steps 3-7 until the following criteria is achieved:

$$\text{error}\ \bar{I}_p = \text{Max}\left|\frac{\bar{I}_p^n - \bar{I}_p^{n-1}}{\bar{I}_p^n}\right| \leq 10^{-6}\ \text{and,}\ \ \text{error}\Theta_p = \text{Max}\left|\frac{\Theta_p^n - \Theta_p^{n-1}}{\Theta_p^n}\right| \leq 10^{-6}.$$

Where, n and n-1 denote the current and previous iteration value of the parameters, respectively. In order to solve the energy equation, Gauss-Seidel iteration method is used. [19].

5. VALIDATION

To show the accuracy of the present method, the algorithm is first tested for a simple geometry, namely a square enclosure. Then, some test problems are considered to show the performance and applicability of the present method to solve radiative problem in irregular geometries.

a) Conduction-radiation

Case 1:

The first test case deals with the combined conductive radiative heat transfer in a cavity of infinite length with a square cross-section containing an absorbing, emitting and non-scattering medium with conduction-radiation parameter ($N_{cr} = \frac{k_0 \beta}{4\sigma T_{ref}^3}$ = 0.25, 2.5, 25.0), optical thickness (βL_* =1.0) and black walls. The left wall is at dimensionless temperature, (Θ_{left}=1.0) and for other walls (Θ_{other} =0.5). Besides, constant thermal conductivity ($\lambda = 0.0$) has been made. Geometry of this problem is shown in Fig. 3a. Figure 3b shows the variation of mid-plane temperature (Y =0.5) distribution. The results of the present study match with the findings of Kim et al. [11]. They solved the same problem by using the DOM with S_4 angular quadrature, diamond spatial scheme and uniform grid for the radiative part of the problem, while the conductive term is discretized using the central difference scheme. In this problem, we have used S_8 approximate with ($N_x \times N_y$ = 40×40) spatial mesh and step scheme. Figure 3b shows good consistency between the present numerical results with theoretical findings of Kim et al [11].

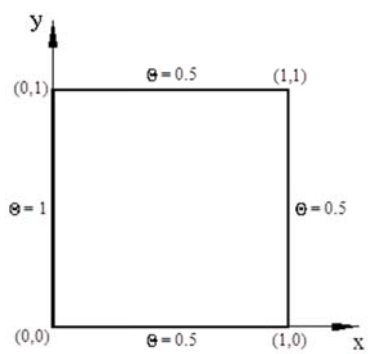


Fig. 3a. Geometry of case 1

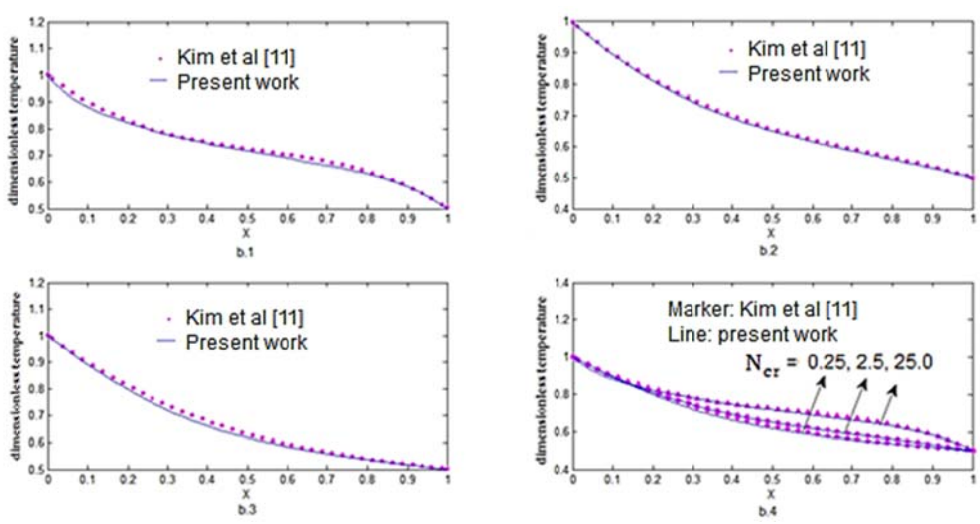


Fig. 3b. Variation of mid-plane temperature (Y=0.5) for case 1; (b.1:N_{cr}=25.0), (b.2: N_{cr} =2.5),(b.3: N_{cr} =0.25),(b.4: effect of N_{cr}), comparison with the results of Kim et al [11]

b) The blocked-off method

Case 2:

For the second problem, we consider radiative heat transfer from four similar solid blocks, placed symmetrically in a square enclosure as shown in Fig. 4a.The relative dimension of the blocks and their placement inside the computational domain with respect to that of the enclosure, are also shown in Fig. 4a.The free space within the domain has been assumed to be filled with a gas medium that has been considered to be absorbing and emitting. All four of the boundaries of the computational domain have been considered to be black ($\varepsilon = 1.0$), and maintained at a constant temperature of (300^{o} K) while the inner solid blocks are assumed to be at a uniform higher temperature of (600^{o} K). The absorption coefficient of the gas ($\kappa = 1.0\ m^{-1}$) and radiative equilibrium has been assumed within the enclosure. In this problem, we have used S_8 approximate with ($N_x \times N_y$ = 40×40) spatial mesh and step scheme.

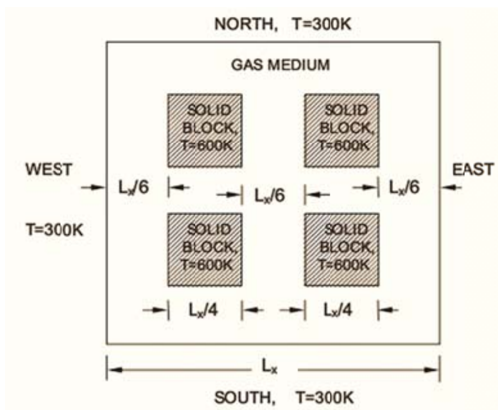


Fig. 4a. Computational domain considered for validation of pure radiation problem (case 2) four similar solid blocks placed symmetrically

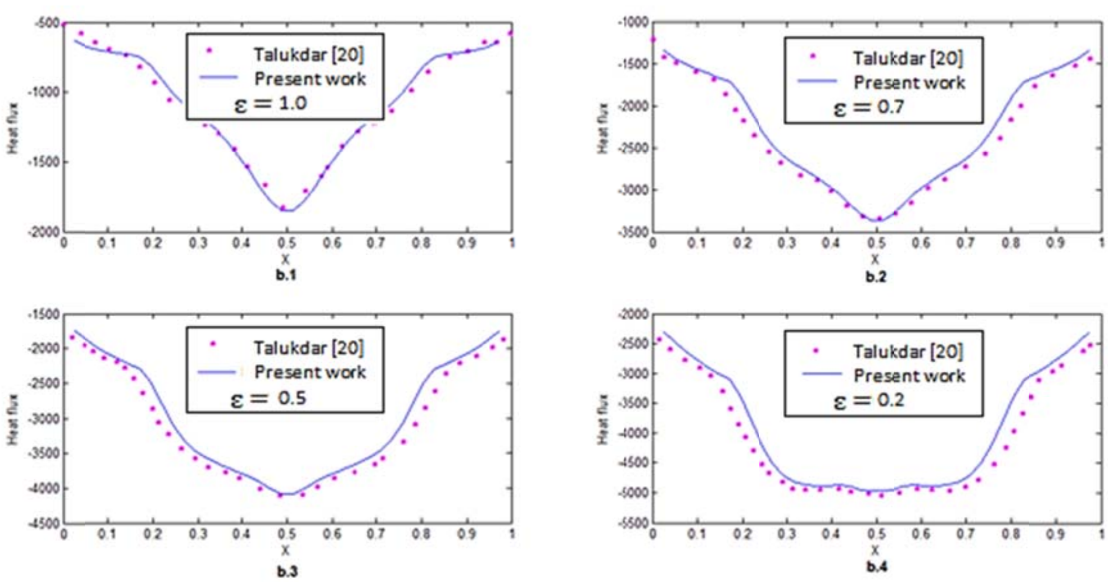


Fig.4-b. Radiative heat flux along the south boundary of the enclosure for different emissivities (b.1:ε =1.0), (b.2: ε =0.7), (b.3: ε =0.5), (b.4: ε =0.2), comparison with the results of Talukdar et al. [20]

The comparisons of the radiative heat fluxes along the south wall of the enclosure for varying emissivities (ε = 0.2, 0.5, 0.7 and 1.0) with theoretical findings by Talukdar et al [20] are depicted in Fig. 4b. They solved this problem by using a new methodology in a complex porous and blocked-off method. It is seen from this figure that an excellent agreement is obtained for situations.

6. RESULTS

The following theoretical results are about combined conductive-radiative heat transfer in the enclosure shown in Fig. 5, in which a T shape enclosure with gray surfaces and one heat source with triangle shape is considered. The boundary walls are kept at specified temperatures as follows (Table 1):

Table 1. Surface temperatures for T- shape enclosure

Wall number	1	2 and 3	4 and 5	6 and 7	8
Temperature(k)	400	600	800	900	1000

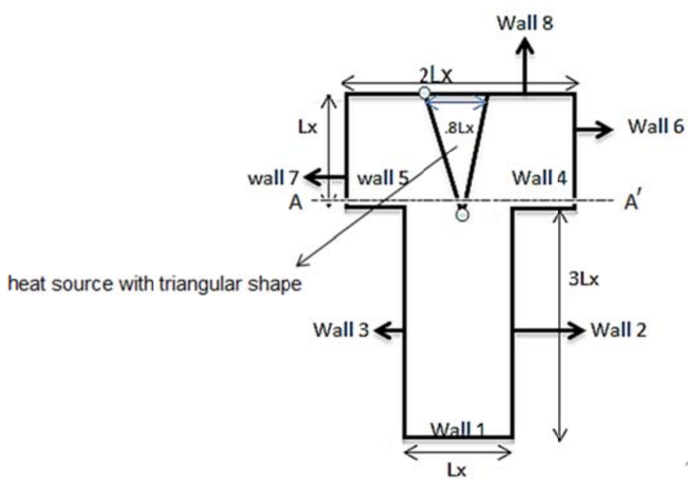


Fig. 5. Geometry of test problem

In this problem, we investigate the effect of variable thermal conductivity on the thermal characteristics of radiative-conductive system.

For all of the test cases, (ω=0.5), ($\beta = 1.0$), ($T_{ref} = T_{wall8}$) and the power of heat source is considered $10^6(\frac{w}{m^3})$. Besides, S_8 approximate is used along the DOM with ($N_x \times N_y$ = 40×80) spatial mesh.

If in combined conductive - radiative heat transfer, the thermal conductivity of participating media is assumed to be variable as a function of temperature according to Eq. (2), there are two related main non-dimensional parameters including (N_{cr}) and (λ). In Fig. 6, the variation of thermal conductivity as a function of temperature for air is plotted. In this figure, the exact values for air thermal conductivity is taken based on the following formula, according to Ref [21]:

$$k(T) = 7.632 \times 10^{-2} + 9.705\times 10^{-5}(T - 1000) \quad (26)$$

Accordingly:

$$N_{cr} = \frac{k_0\beta}{4\sigma T_{ref}^3} = 3.3 \times 10^{-4}$$

$$\lambda = \frac{\alpha' T_{ref} N_{cr}}{k_0} = 4.4 \times 10^{-4}$$

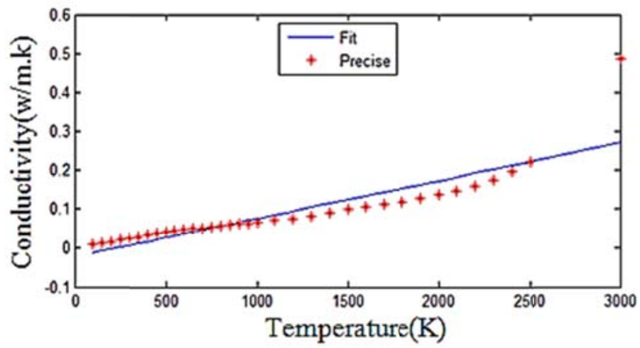


Fig. 6. Variation of thermal conductivity with temperature

a) Effect of conductivity-temperature coefficient

In this section, the effect of variation of thermal conductivity on thermal behavior of the combined conductive-radiative system is studied. In Fig.7, the distributions of gas temperature along the AA′ section which is depicted in Fig. 5 are plotted for four different values of the parameter (λ). In the computation of this figure, it is assumed that all surrounding walls are black by considering (N_{cr}=0.00033) and assuming isotropic scattering participating media. It is seen that conductivity-temperature coefficient has a slight effect on temperature distribution, considering the fact that the value of gas maximum and minimum temperatures increase with increasing in conductivity-temperature coefficient. Besides, in order to have much clearer figure for demonstrating the effect of parameter λ on thermal behavior of the system, the temperature distributions in AA′ section for the maximum and minimum values of conductivity-temperature coefficient are compared to each other in Figure 8. It can be seen that temperature distribution has different trends near the torch because of different values of λ and also the maximum difference of about 4% is depicted in Fig. 8 for the temperature computation at λ= 0.00044, and λ= -0.0022.

Prediction of temperature distribution in the chamber when λ is not zero is difficult. To clear the subject in Fig. 9, variation of heat conductivity is shown in section AA′. It can be seen that as the value of λ increases, the thermal conductivity near the wall decreases and around the torch increases. The interesting point in this figure is that the lowest thermal conductivity for (λ=-0.0022) occurs at the torch. In other words, by decreasing the value of λ, conductive heat transfer near the wall increases and it causes the temperature to be closer to the wall temperature, while around the torch, the opposite behavior is seen.

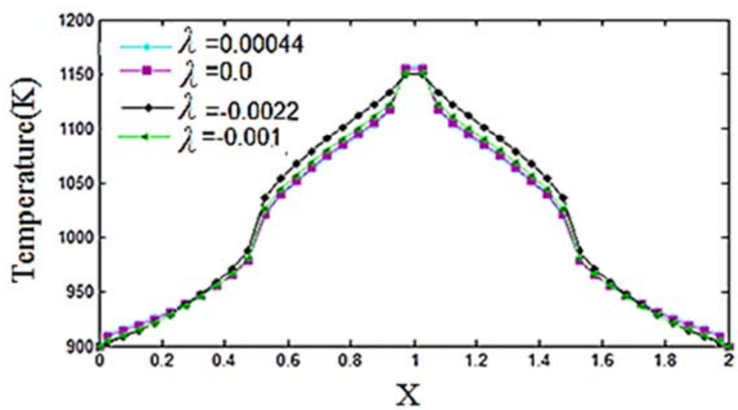


Fig. 7. The effect of conductivity-temperature coefficient on the distribution temperature in the section AA′, for ($N_{cr} = 0.00033, \varepsilon_w = 1.0, \beta = 1.0$, ω=0.5)

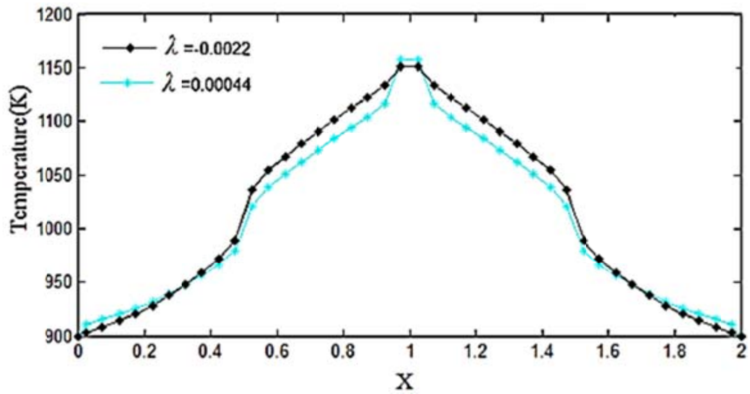


Fig. 8. Temperature distribution in the section AA′, for two different values of λ, ($N_{cr} = 0.00033, \varepsilon_w = 1.0, \beta = 1.0$, ω=0.5)

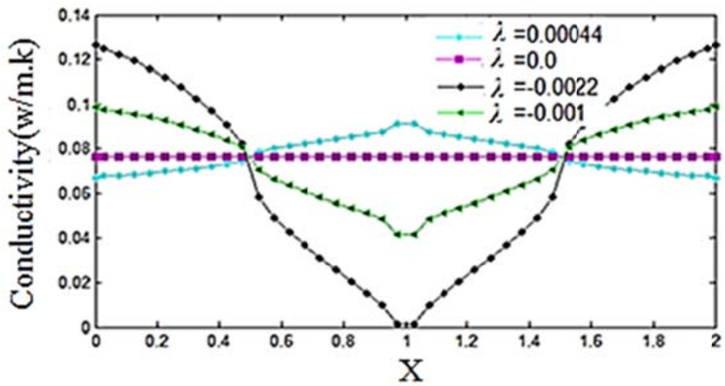


Fig. 9. Variation of thermal conductivity in the section AA′

Figure 10 depicts the effect of parameter λ on distribution of conductive heat flux fraction ($\frac{q_c}{q_c+q_r}$) along the bottom wall. It is revealed from this figure that conductive heat flux has an almost uniform distribution over the bottom wall, especially for high value of parameter λ, such that for negative value of conduction-temperature coefficient, conductive heat flux in the region near to the surrounding walls increases. Moreover, it is seen from Fig. 10 that high rate of conductive heat flux takes place by decreasing the value of parameter λ, especially on the edges of the bottom wall. However, in all cases, the contribution of the conduction heat transfer is less than 25%.

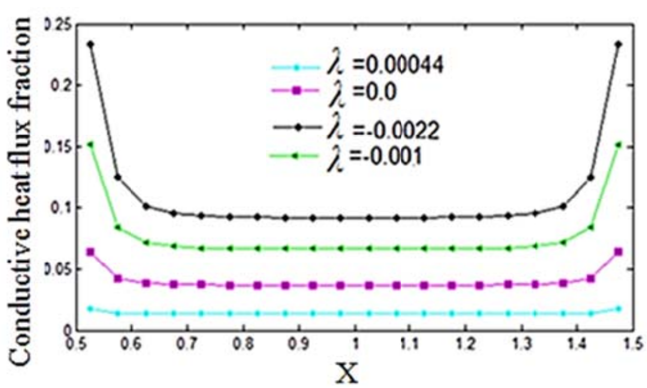


Fig. 10. The effect of conductivity-temperature coefficient on the conductive heat flux fraction over the bottom wall, for ($N_{cr} = 0.00033, \varepsilon_w = 1.0, \beta = 1.0$, ω=0.5)

b) Effect of conduction-radiation parameter

The effects of conduction-radiation parameter, (N_{cr}), on distribution of conductive heat flux fraction over the bottom wall are shown in Fig. 11. In this case, the whole walls are black, scattering is isotropic and also, (λ) is constant (λ= 4.4×10^{-4}). It is seen that (N_{cr}) has significant effect on conduction heat flux, such that for small value of (N_{cr}) in which the radiation heat transfer is dominant, there is almost uniform distribution of conductive heat flux over the bottom wall with very small value, such that conduction heat flux has increasing trend with increasing in (N_{cr}). By focusing on Fig. 12 in which the variations of absolute total heat flux on the bottom wall are shown, it is seen that the same trend as was observed before in Fig. 11 for conductive heat flux also exists for absolute total heat flux over the bottom wall.

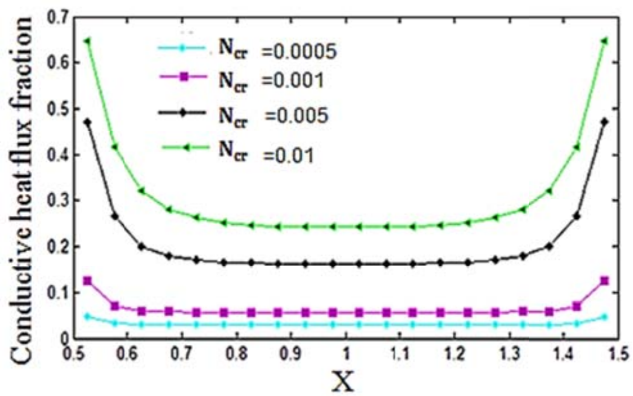


Fig.11.The effect of Conduction-radiation parameter on the conductive heat flux fraction over the bottom wall, for ($\lambda = 0.00044, \varepsilon_w = 1.0, \beta = 1.0$, ω=0.5)

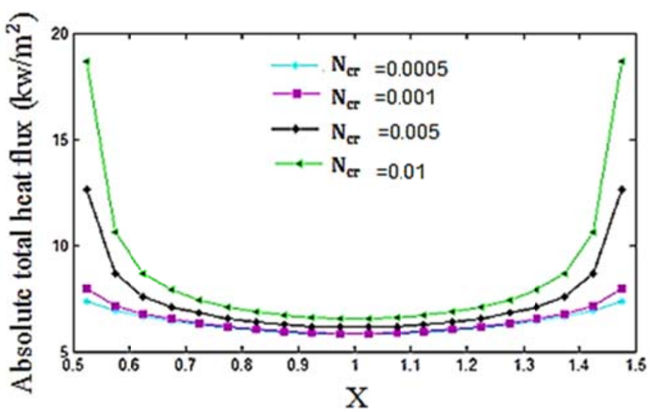


Fig.12. The effect of Conduction-radiation parameter on the absolute total heat flux over the bottom wall, for ($\lambda = 0.00044, \varepsilon_w = 1.0, \beta = 1.0$, ω=0.5)

The temperature distribution within enclosure is plotted in Fig. 13, in which four different values of conduction radiation parameter were used in the computations. It is seen that at small value of (N_{cr}) which is due to radiation dominance in a participating media, the value of maximum temperature inside the enclosure increases and the effect of heat source on the whole domain of the media becomes more clear. This fact can be seen easily if one compares Figs. 13a and d with each other. It is observed that, with increasing in the value of conduction radiation parameter, near the boundaries, the temperature inside the media becomes closer to the wall temperature due to increasing in conduction heat flux on the boundary walls.

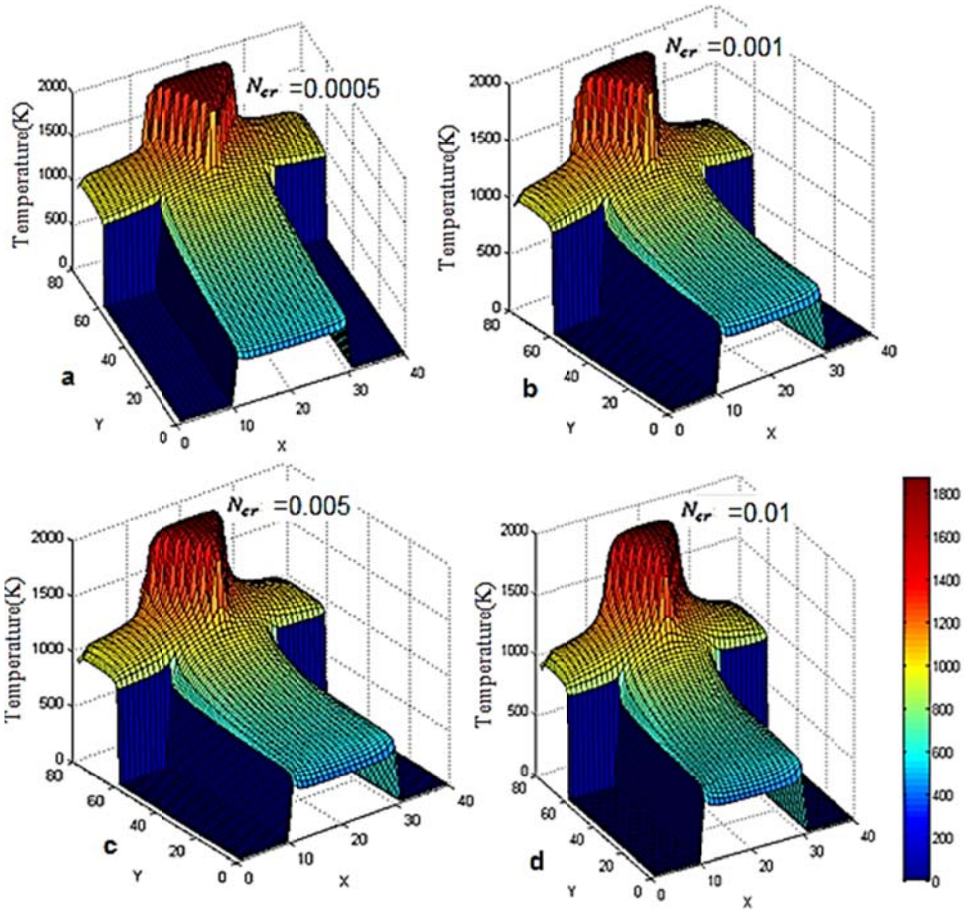


Fig. 13. The effect of Conduction-radiation parameter on the temperature distribution within enclosure. (a):N_{cr}=0.0005,(b):N_{cr}=0.001, (c):N_{cr}=0.005,(d):$N_{cr} = 0.01$, for ($\lambda = 0.00044, \varepsilon_w = 1.0, \beta = 1.0$, ω=0.5)

7. CONCLUSION

The DOM and FVM with blocked-off method were used to analyze combined conduction –radiation heat transfer in a T shape enclosure with one triangular shape heat source within absorbing, emitting and scattering medium with variable thermal conductivity. The RTE and energy equations were solved with DOM and FVM, respectively. The results of constant thermal conductivity and isotropic scattering have a very good agreement with those available in the literature. The effect of variable thermal conductivity on temperature distributions and conductive-radiative heat flux was studied.

The results of this study can be summarized as follows:

-In combined conduction and radiation heat transfer, there is always a contrast between these two modes of heat transfer. Conduction heat transfer is dominant near the boundaries and radiation heat transfer is important near the heat source zone. With increasing the relative contribution of heat conduction, the temperature of the flame and boundaries penetrates further into the environment.

-Variation of thermal conductivity coefficient with temperature causes many difficulties in prediction of temperature distribution and heat flux in the chamber. However, due to the small value of this quantity for air, the thermal behavior of environment was not affected by this parameter, but in high temperature systems, the effect of this parameter will be more important.

-According to the results, combining DOM with blocked-off method demonstrates a simple numerical tool with suitable accuracy to analyze coupled conduction – radiation heat transfer problems in 2-D irregular geometries.

NOMENCLATURE

A	surface area (m^2)
G	incident radiation (W/m^2)
r	position(m)
I	radiation intensity (W/m^2 sr)
$\bar{I}$	dimensionless radiation intensity
K	thermal conductivity (W/mK)
k_0	reference thermal conductivity (W/mK)
L	length (m)
M	number of discrete directions
$\vec{n}_w$	outward unit vector normal to the wall
N_{cr}	conduction-radiation parameter
q	heat flux (W/m^2)
Q	dimensionless heat flux
$\dot{Q}'''$	heat source per unit volume (W/m^3)
$\vec{s}$	direction vector (m)
S	source term (W/m^2)
T	temperature(K)
V	volume (m^3)
w	weight of angular quadrature
x, y, z	spatial coordinate (m)
X, Y, Z	dimensionless coordinate

Greek symbols

α'	variable thermal conductivity parameter (W/mk^2)
λ	non-dimensional conductivity-temperature coefficient
β	extinction coefficient (m^{-1})
γ	spatial differencing weights
ε_w	emissivity of the surface
Θ	dimensionless temperature
κ	absorption coefficient (m^{-1})
ξ, η	x- and y- direction cosines
ρ_w	reflectivity of the surface
ρ_f	medium density
σ	Stefan-Boltzmann constant = $5.6704 \times 10^{-8} W/m^2K^4$
σ_s	scattering coefficient (m^{-1})
τ	optical thickness
ω	scattering albedo
Ω	solid angel (sr)

Subscripts

ref	reference quantity
b	black body
out	exit
in	input
m	discrete direction
p	nodal point
r	radiation
x, y	coordinates

Superscripts

in	incoming direction
m	discrete direction
n	iteration level

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