IJST, Transactions of Mechanical Engineering, Vol. 38, No. M2, pp 337-349
Printed in The Islamic Republic of Iran, 2014

VIBRATION DISSIPATION OF A SHELL CONTAINING FLUID*

A. NAJAFI[1**] AND M. AHMADZADEH[2]

[1]Faculty of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, I. R. of Iran
[2]Young Researchers and Elite Club, Shiraz Branch, Islamic Azad University, Shiraz, I. R. of Iran
Email: najafi@kntu.ac.ir

Abstract– In this article, mechatronic control of a shell vibration containing fluid (partially filled with a fluid) has been addressed. Simple actuators are utilized to achieve boundary stabilization. These actuators apply linear boundary feedbacks consisting of forces and moments from the boundaries of the shell to dissipate the vibrations. In this research, the fluid has free surface and without attaching any sensors or actuators in the fluid domain, the vibration suppression is obtained. This research utilizes semigroup approach and LaSalle invariant set theorem to prove the boundary stabilization.

Keywords– Semigroups techniques, asymptotic boundary stabilization, elastic shell, fluid-structure interaction problems

1. INTRODUCTION

Nowadays, flexible materials such as beams, plates and shells are widely used in many industrial processes. For example, in airplane wings, satellite antennas, flexible manipulators, tanks containing fluids, pressure vessels, etc. Petrochemical applications, Water industries, Aeronautics and Aerospace are the most important fields of engineering, where the use of complex flexible materials such as shells is a common practice.

Boundary stabilization of a shell with a fluid has been studied. Such problems appear commonly in practice [1]. Therefore, many authors have considered the vibrations of the structures in contact with a fluid [2; 3; 4].

Vibration is well known for its ability to cause discomfort, fatigue and destruction. In addition, low rigidity and small structural damping of structures may amplify vibrations.

Vibration suppression of flexible bodies which may or may not be in contact with a fluid has been considered by many authors [5; 6].

In this study, the shell containing fluid was studied. Some researchers have been concerned with the fluid-structure stabilization problems, [7; 1]. In these studies, the fluid has no free surface; but actually, in most fluid-structure problems such as dams, large containers, the fluid has at least one free surface. In addition, none of the aforementioned works considered shell as their structure. Therefore, in this work, we study the stabilization problem of the vibrating shell containing fluid.

We adopt semigroup techniques and Lyapunov theorem. Semigroup techniques are used to demonstrate the well-posedness of the system.

*Received by the editors November 4, 2013; Accepted February 3, 2014.
**Corresponding author

The remainder of this article is arranged as follows: in section 2, the dynamics of a shell and fluid are presented. Section 3 is devoted to well-posedness and boundary stabilization proof of the fluid-structure problem. Section 4 is devoted to the simulations and the conclusion is presented in Section 5.

2. GOVERNING EQUATIONS OF MOTION

a) Fluid domain

The governing equation for the Newtonian barotropic fluid with low velocity can be simplified from the Navier-Stokes equation to the wave equation [8]. The related equations are listed below,

$$\begin{cases} c^2 \Delta\phi = \phi,_{tt} & \text{in} \quad \Theta \\ \rho_0 \phi,_t = -p(x,\theta,t) & \text{in} \quad \Omega \\ \rho_0 \phi,_{tt} + \rho_0 g \phi,_n + p_e,_t = 0 & \text{in} \quad \Omega_2 \\ \phi,_n = 0 & \text{in} \quad \Omega_3 \end{cases} \tag{1}$$

where ρ_0, R are fluid density and shell radius respectively and also Ω, Ω_2 and Ω_3 are defined as follows

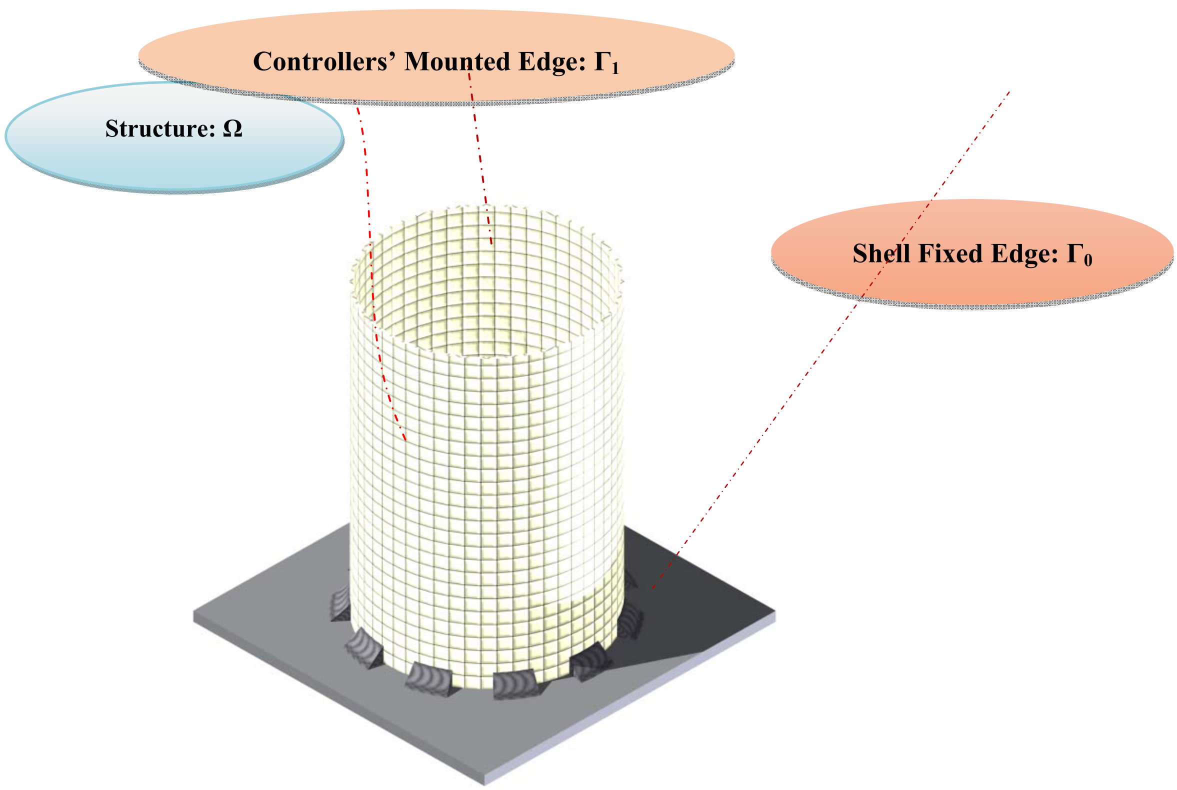

Fig. 1. Schematic view of the general fluid-structure problem

1) The wet surface or the fluid structure interface, Ω: (Fig. 1.)

This is the most essential part of the fluid boundary. The motion of the structure and the normal component of the fluid motion coincide, that is [9]:

$$\mathbf{v}_f.\mathbf{n} = \mathbf{v}_s.\mathbf{n} \tag{2}$$

where $\mathbf{v}_f$ is the fluid velocity and $\mathbf{v}_s$ is the structure velocity.
In this boundary the following equation can be attained [9]:

$$\rho_0 \phi,_t = -p(x,\theta,t) \tag{3}$$

where p is the hydrodynamic pressure.

2) A free surface with prescribed external pressure where we allow the linearized (gravitational) waves, Ω_2 see Fig. 1., [9]:

$$\rho_0 \phi,_{tt} + \rho_0 g \phi,_n + p_e,_t = 0 \tag{4}$$

3) Fixed surface with prescribed external pressure, Ω_3, see Fig. 1., [9]:

$$\phi,_n = 0 \tag{5}$$

where, $\phi(x,y,z,t)$ is the velocity potential. It means that $\mathbf{v} = \nabla\phi$ and c is the sound speed in the fluid.

b) Structure domain

The governing equations of a shell with external pressure $p(x,\theta,t)$ can be written as follows [9]:

$$\begin{aligned}
\rho h \ddot{u} &= \frac{\partial N_{xx}}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} \\
\rho h \ddot{v} &= \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{1}{R} Q_{\theta z} \\
\rho h \ddot{w} &= \frac{\partial Q_{xz}}{\partial x} + \frac{1}{R}\frac{\partial Q_{\theta z}}{\partial \theta} - \frac{1}{R} N_{\theta\theta} + p(x,\theta,t) \\
Q_{xz} &= \frac{\partial M_{xx}}{\partial x} + \frac{1}{R}\frac{\partial M_{x\theta}}{\partial \theta} \\
Q_{\theta z} &= \frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial M_{\theta\theta}}{\partial \theta}
\end{aligned} \tag{6}$$

where u, v, w and ρ represent three displacements and shell density respectively (Fig. 2).

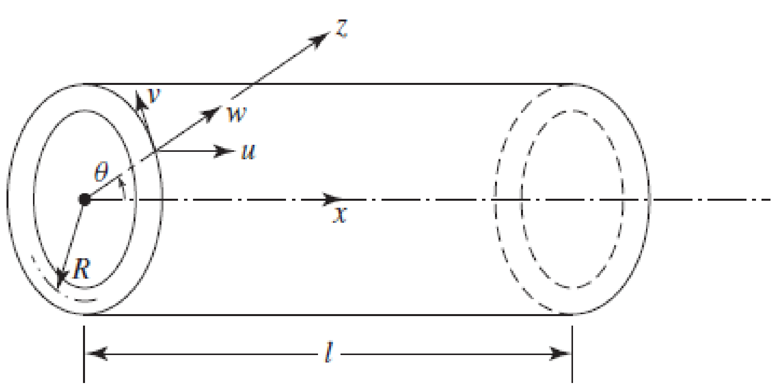

Fig. 2. Shell coordinates

The strain-displacement relations for the cylindrical shell are as follows [10]:

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} \\
\varepsilon_{\theta\theta} &= \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{w}{R} \\
\gamma_{x\theta} &= \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} \\
k_{xx} &= -\frac{\partial^2 w}{\partial x^2} \\
k_{\theta\theta} &= \frac{1}{R}\left(\frac{1}{R}\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right) \\
\tau &= \frac{1}{R}\frac{\partial v}{\partial x} - \frac{2}{R}\frac{\partial^2 w}{\partial x \partial \theta}
\end{aligned} \tag{7}
$$

The relations between resultant forces / moments and strains are as follow:

$$
\begin{bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \end{bmatrix} =
\begin{bmatrix}
\frac{Eh}{(1-\nu^2)} & \frac{Eh\nu}{(1-\nu^2)} & 0 & 0 & 0 & 0 \\
\frac{Eh\nu}{(1-\nu^2)} & \frac{Eh}{(1-\nu^2)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{Eh}{2(1+\nu)} & 0 & 0 \\
0 & 0 & 0 & \frac{Eh^3}{12(1-\nu^2)} & \frac{Eh^3\nu}{12(1-\nu^2)} & 0 \\
0 & 0 & 0 & \frac{Eh^3\nu}{12(1-\nu^2)} & \frac{Eh^3}{12(1-\nu^2)} & 0 \\
0 & 0 & 0 & \frac{Eh^3}{12(1-\nu^2)} & 0 & 0
\end{bmatrix}
\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{\theta x} \\ k_{xx} \\ k_{\theta\theta} \\ \tau \end{bmatrix} \tag{8}
$$

We utilize the following notations for the rest of the paper

$$
S = \begin{bmatrix}
\frac{Eh}{(1-\nu^2)} & \frac{Eh\nu}{(1-\nu^2)} & 0 & 0 & 0 & 0 \\
\frac{Eh\nu}{(1-\nu^2)} & \frac{Eh}{(1-\nu^2)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{Eh}{2(1+\nu)} & 0 & 0 \\
0 & 0 & 0 & \frac{Eh^3}{12(1-\nu^2)} & \frac{Eh^3\nu}{12(1-\nu^2)} & 0 \\
0 & 0 & 0 & \frac{Eh^3\nu}{12(1-\nu^2)} & \frac{Eh^3}{12(1-\nu^2)} & 0 \\
0 & 0 & 0 & \frac{Eh^3}{12(1-\nu^2)} & 0 & 0
\end{bmatrix}, \; E = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{\theta x} \\ k_{xx} \\ k_{\theta\theta} \\ \tau \end{bmatrix} \tag{9}
$$

where $p(x,\theta,t)$ is the external transverse force distribution (hydrodynamic pressure due to fluid loading) on the shell, *h* is the thickness of the shell, and *E* and *v* are Young's modulus and poisson ratio. It should be noted that Ω is a bounded simple region.

The structure boundaries are as follows:

1) Fixed edge, Γ_0, in this boundary, there is no structure motion, see Fig. 1.

2) Free edge (controllers' mounted edge), Γ_1, in this boundary, the controllers are attached see Fig. 1.

3. STABILIZATION OF SHELL FILLED WITH THE FLUID

In this section, we consider the stabilization problem of the shell vibrations. For this purpose, first, the following definitions will be used.

The inner product on the space $\mathbf{H} = H^2_{\Omega_3}(\Theta)\times L^2(\Theta)\times(H^2_{\Gamma_0}(\Omega)\times L^2(\Omega))^3$ will be presented as

$$\begin{aligned} < X,Y >_{\mathbf{H}} &= \int_\Theta [\frac{\rho_0}{2c^2}x_2y_2 + \frac{\rho_0}{2}\Pi(x_1,y_1)]d\theta \\ &+\int_\Omega [(\rho_0/2g)x_2y_2 + (\rho h/2)(x_4y_4 + x_6y_6 + x_8y_8) + \Lambda(X,Y)]d\Omega \end{aligned} \tag{10}$$

where

$$X,Y\in\mathbf{H}\ ,X=(x_1,\,x_2,\,x_3,\,x_4,\,x_5,\,x_6,\,x_7,\,x_8)\,,Y=(y_1,\,y_2,\,y_3,\,y_4,\,y_5,\,y_6,\,y_7,\,y_8)\,,$$
$$H^2_{\Omega_3}(\Theta)=\{x_1:x_1\in H^2(\Theta):\partial x_1/\partial n=0\big|_{\Omega_3}\}\ ,\ H^1_{\Omega_3}(\Theta)=\{x_1:x_1\in H^1(\Theta):\partial x_1/\partial n=0\big|_{\Omega_3}\}$$

and

$$\mathbf{H}^2_{\Gamma_0}(\Omega)=\{(x_3,\,x_5,\,x_7):(x_3,\,x_5,\,x_7)\in(H^2(\Omega))^3: x_i=0\big|_{\Gamma_0},\partial x_i/\partial n\big|_{\Gamma_0}=0,\ i=1,2,3\}\,;$$

also, the following relations hold

$$\begin{cases}\Pi(\kappa_1,\kappa_2)=\kappa_1,_x\,\kappa_2,_x+\kappa_1,_y\,\kappa_2,_y+\kappa_1,_z\,\kappa_2,_z \\ \Lambda(X,Y)=(1/2)\,X^T S\,Y\end{cases} \tag{11}$$

It should be noted that $\Lambda(X,X)$ and $\Pi(\kappa,\kappa)$ take the roles of the strain energy of the shell and the fluid respectively and therefore must be nonnegative.

The shell governing equations and related boundary conditions are as follows:

$$\left\{\begin{array}{lll}\rho h\ddot{u}=\dfrac{\partial N_{xx}}{\partial x}+\dfrac{1}{R}\dfrac{\partial N_{x\theta}}{\partial\theta} & \text{in} & \Omega \\ \rho h\ddot{v}=\dfrac{\partial N_{x\theta}}{\partial x}+\dfrac{1}{R}\dfrac{\partial N_{\theta\theta}}{\partial\theta}+\dfrac{1}{R}Q_{\theta z} & \text{in} & \Omega \\ \rho h\ddot{w}=\dfrac{\partial Q_{xz}}{\partial x}+\dfrac{1}{R}\dfrac{\partial Q_{\theta z}}{\partial\theta}-\dfrac{1}{R}N_{\theta\theta}+p(x,\theta,t) & \text{in} & \Omega \\ u=v=w=\partial u/\partial n=\partial v/\partial n=\partial w/\partial n=0 & \text{in} & \Gamma_0 \\ N_{xx}=U_1,\ N_{x\theta}=U_2,\,Q_{xz}=U_3 & \text{in} & \Gamma_1 \\ M_{xx}=U_4,\,M_{x\theta}=U_5 & \text{in} & \Gamma_1\end{array}\right\} \tag{12}$$

For this problem, our main intention is to show that the system (12) under related boundary feedback $U_i=-k_f x_{2(i+1)}$ and $U_4=-k_m w,_{xt}\,, U_5=-k_m R(v,_t - w,_{t\theta})$ is well-posed and asymptotically stable. Note that $\partial\Omega=\Gamma=\Gamma_0\cup\Gamma_1$.

To analyze the system using the notions of the linear operators, the following notations are utilized

$$A\,X=\begin{pmatrix} x_2 \\ c^2\Delta\, x_1 \\ x_4 \\ \dfrac{1}{\rho\, h}(\dfrac{\partial N_{xx}}{\partial x}+\dfrac{1}{R}\dfrac{\partial N_{x\theta}}{\partial \theta}) \\ x_6 \\ \dfrac{1}{\rho\, h}(\dfrac{\partial N_{x\theta}}{\partial x}+\dfrac{1}{R}\dfrac{\partial N_{\theta\theta}}{\partial \theta}+\dfrac{1}{R}Q_{\theta z}) \\ x_8 \\ \dfrac{1}{\rho\, h}[\dfrac{\partial Q_{xz}}{\partial x}+\dfrac{1}{R}\dfrac{\partial Q_{\theta z}}{\partial \theta}-\dfrac{1}{R}N_{\theta\theta}+p(x,R,\theta)] \end{pmatrix} \tag{13}$$

The state space representation of the system of Eq. (12) is:

$$\left\{\begin{array}{l} \dot{X}=A\;X \\ u=v=w=0,\;\partial u/\partial n=\partial v/\partial n=\partial w/\partial n=0 \qquad \text{in} \quad \Gamma_0 \\ U_1=-k_f\,u,_t\,,U_2=-k_f\,v,_t\,,U_3=-k_f\,w,_t\,,U_4=k_m\,w,_{tx}\,,U_5=-k_m R(v,_t-w,_{t\theta}),\;\text{in} \quad \Gamma_1 \\ \rho_0\,\phi,_t=-p \qquad \text{in} \quad \Omega \\ \rho_0\,\phi,_{tt}+\rho_0\,g\,\phi,_n=0 \qquad \text{in} \quad \Omega_2 \\ \phi,_n=0 \qquad \text{in} \quad \Omega_3 \\ X(0)=X_0 \end{array}\right\} \tag{14}$$

At first, it will be shown that the operator *A* with the following domain is a dissipative operator.

$$\begin{gathered} D(A)=\{(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8)\big|\,x_1\in H^2(\Theta)\cap H^1_{\Omega_3}(\Theta),\;x_2\in H^1_{\Omega_3}(\Theta), \\ x_3,\,x_5,\,x_7\in H^2_{\Gamma_0}(\Omega)\cap H^4(\Omega),\,x_4,\,x_6,\,x_8\in H^2_{\Gamma_0}(\Omega)\;\text{ such that }\;\rho_0\,x_2\big|_{\Omega}=-p\} \end{gathered} \tag{15}$$

where $H^4_{\Gamma_0}(\Omega)=\{x:x\in H^4(\Omega):x=0\big|_{\Gamma_0},\partial\,x/\partial n\big|_{\Gamma_0}=0\}$ and $H^2_{\Omega_3}(\Theta)=\{x:x\in H^2(\Theta),\,x=0\big|_{\Omega_3}\}$ and $D(A)$ stand for domain of operator *A*.

Lemma 1. (*I* - *A*) is an accretive operator and its inverse exists and is compact.

Proof. We start from the fact that the total mechanical energy of the systems is equal to the following inner product $V(t)=<X\,,X>$, therefore

$$\dot{V}(t)=2<X,\dot{X}>=2<X\,,AX> \tag{16}$$

For the rest of the proof, it will be shown that the following relation holds for the coupled fluid-structure system

$$\dot{V}=-\,[\int_0^{2\pi}\{u,_t^2+v,_t^2+w,_t^2+w,_{xt}^2+(v,_t-w,_{t\theta})^2\}]_0^l\;d\theta \tag{17}$$

where $V=V_{sh}+V_f$ is the total energy of the coupled system.

$$V_{sh}=\frac{1}{2}\int_0^{2\pi}\int_0^l\{E^T S\,E+\rho h(u,_t^2+v,_t^2+w,_t^2)\}R\,d\theta\,dx \tag{18}$$

Based on the symmetry of the matrix *S*, one can easily infer that $\dot{E}^T S\, E = E^T S\, \dot{E}$ and therefore

$$\dot{V}_{sh} = \int_0^{2\pi}\int_0^{l} \{\dot{E}^T S\, E + \rho h\,(u,_t\, u,_{tt} + v,_t\, v,_{tt} + w,_t\, w,_{tt}\,)\} R\, d\theta\, dx \tag{19}$$

Utilizing the governing Eq. (12), the Eq. (19) turns to

$$\begin{aligned}\dot{V}_{sh} = \int_0^{2\pi}\int_0^{l} &\{N_{xx} u,_{xt} + \frac{N_{\theta\theta}}{R}(v,_{\theta t} + w,_{tx}\,) + N_{\theta x}(v,_{xt} + \frac{u,_{\theta t}}{R})\\ &+ \frac{M_{\theta\theta}}{R}(\frac{v,_{\theta t}}{R} - w,_{\theta\theta t}\,) + \frac{M_{x\theta}}{R}(v,_{xt} - 2w,_{x\theta t}\,)\\ &- w,_{xxt}\, M_{xx} + N_{xx},_x\, u,_t + \frac{N_{x\theta}}{R} u,_t + N_{x\theta},_x\, v,_t\\ &+ \frac{N_{\theta\theta},_\theta}{R} v,_t + \frac{Q_{\theta z}}{R} v,_t + Q_{xz},_x\, w,_t + \frac{Q_{\theta z},_\theta}{R} w,_t\\ &- \frac{N_{\theta\theta}}{R} w,_t + p\, w,_t\,\} R\, d\theta\, dx\end{aligned} \tag{20}$$

Rearranging and collecting appropriate terms leads to

$$\begin{aligned}\dot{V}_{sh} = \int_0^{2\pi}\int_0^{l} &\{(N_{xx} u,_t\,),_x + \frac{1}{R}(N_{x\theta} u,_t\,),_\theta + (N_{x\theta} v,_t\,),_x + \frac{1}{R}(N_{\theta\theta} v,_t\,),_\theta\, (\frac{\partial^2 u}{\partial t \partial x})\\ &+ \frac{M_{\theta\theta}}{R}(\frac{v,_{\theta t}}{R} - w,_{\theta\theta t}\,) + \frac{M_{x\theta}}{R}(v,_{xt} - 2w,_{x\theta t}\,)\\ &- w,_{xxt}\, M_{xx} + \frac{Q_{\theta z}}{R} v,_t + Q_{xz},_x\, w,_t + \frac{Q_{\theta z},_\theta}{R} w,_t\\ &+ p\, w,_t\,\} R\, d\theta\, dx\end{aligned} \tag{21}$$

Concerning the following relations

$$\begin{aligned} Q_{\theta z} &= M_{x\theta},_x + \frac{M_{\theta\theta},_\theta}{R}\\ Q_{xz} &= M_{xx},_x + \frac{M_{x\theta},_\theta}{R}\end{aligned} \tag{22}$$

Again, rearranging and collecting appropriate terms in the Eq. (23) one can obtain

$$\begin{aligned}\dot{V}_{sh} = \int_0^{2\pi}\int_0^{l} &\{(N_{xx} u,_t\,),_x + \frac{1}{R}(N_{x\theta} u,_t\,),_\theta + (N_{x\theta} v,_t\,),_x\\ &+ \frac{1}{R}(N_{\theta\theta} v,_t\,),_\theta\, (\frac{\partial^2 u}{\partial t \partial x}) + \frac{M_{\theta\theta}}{R}(\frac{v,_{\theta t}}{R} - w,_{\theta\theta t}\,)\\ &+ \frac{M_{x\theta}}{R}(v,_{xt} - 2w,_{x\theta t}\,) - w,_{xxt}\, M_{xx} + (M_{x\theta},_x + \frac{M_{\theta\theta},_\theta}{R})\frac{v,_t}{R}\\ &+ (M_{xx},_{xx} + \frac{M_{x\theta},_{\theta x}}{R}) w,_t + (M_{x\theta},_{x\theta} + \frac{M_{\theta\theta},_{\theta\theta}}{R})\frac{w,_t}{R} + p\, w,_t\,\} R\, d\theta\, dx\end{aligned} \tag{23}$$

and finally

$$\dot{V}_{sh}=\int_0^{2\pi}\int_0^{l}\{(N_{xx}u,_t),_x+\frac{1}{R}(N_{x\theta}u,_t),_\theta+(N_{x\theta}v,_t),_x+\frac{1}{R}(N_{\theta\theta}v,_t),_\theta+(\frac{M_{x\theta}v,_t}{R}),_x$$
$$+(\frac{M_{\theta\theta}v,_t}{R^2}),_\theta+(M_{xx},_x w,_t-M_{xx}w,_{tx}),_x+\frac{1}{R}(M_{x\theta},_x w,_t-M_{x\theta}w,_{tx}),_\theta \tag{24}$$
$$+\frac{1}{R}(M_{x\theta},_\theta w,_t-M_{x\theta}w,_{t\theta}),_x+p\,w,_t\}\,R\,d\theta\,dx$$

However, for the fluid domain the related energy functional is as follows:

$$V_f=\int_\Theta(\frac{\rho_0}{2c^2}\varphi_t{}^2+\frac{\rho_0}{2}(\nabla\varphi)^2)\,d\Theta+\int_{\Omega_2}\frac{\rho_0}{2g}\varphi_t{}^2d\,\Omega_2 \tag{25}$$

Taking the time derivative leads to

$$\dot{V}_f=\int_\Theta[\frac{\rho_0}{c^2}\varphi_t\,\varphi_{tt}+\rho_0(\nabla\varphi).(\nabla\varphi_t)]\,d\Theta+\int_{\Omega_2}\frac{\rho_0}{g}\varphi_t\,\varphi_{tt}\,d\,\Omega_2 \tag{26}$$

Utilizing the governing Eq. (1) yields

$$\dot{V}_f=\int_\Theta[\rho_0\,\varphi_t\nabla^2\varphi+\rho_0(\nabla\varphi).(\nabla\varphi_t)]\,d\Theta+\int_{\Omega_2}\frac{\rho_0}{g}\varphi_t\,\varphi_{tt}\,d\Omega_2 \tag{27}$$

Concerning the related boundary conditions

$$\rho_0\varphi_t=-p \qquad \text{in} \qquad \Omega \tag{28}$$

$$\varphi_n=-\frac{1}{g}\varphi_{tt} \qquad \text{in} \qquad \Omega_2 \tag{29}$$

$$\varphi_n=0 \qquad \text{in} \qquad \Omega_3 \tag{30}$$

one can easily obtain

$$\begin{aligned}\dot{V}_f&=\int_\Theta\rho_0\nabla.(\varphi_t\nabla\varphi)\,d\Theta+\int_{\Omega_2}\frac{\rho_0}{g}\varphi_t\,\varphi_{tt}d\Omega_2\\&=\int_{\Omega,\,\Omega_2,\,\Omega_3}\rho_0\,\varphi_t\,\varphi_n\,d\Omega+\int_{\Omega_2}\frac{\rho_0}{g}\varphi_t\,\varphi_{tt}\,d\Omega_2\\&=\int_\Omega-p\,\varphi_n d\Omega+\int_{\Omega_2}-\frac{\rho_0}{g}\varphi_t\varphi_{tt}d\Omega_2+\int_{\Omega_2}\frac{\rho_0}{g}\varphi_t\,\varphi_{tt}\,d\,\Omega_2\\&=\int_\Omega-p\,\varphi_n\,d\,\Omega\\&=\int_\Omega-p\,w_t\,d\,\Omega\end{aligned} \tag{31}$$

$$\dot{V}=\dot{V}_{sh}+\dot{V}_f \tag{32}$$

$$\begin{aligned}\dot{V}=[\int_0^{2\pi}\{N_{xx}u,_t+N_{x\theta}v,_t-M_{xx}w,_{xt}+\frac{M_{x\theta}}{R}(v,_t-w,_{t\theta})+Q_{xz}w,_t\}]_0^l\,d\theta\\ \int_0^{2\pi}\int_0^{l}p\,w_t\,dx\,d\theta+\int_\Omega-p\,w_t\,d\,\Omega\end{aligned} \tag{33}$$

which yields

$$\dot{V}=[\int_0^{2\pi}\{N_{xx}u,_t+N_{x\theta}v,_t-M_{xx}w,_{xt}+\frac{M_{x\theta}}{R}(v,_t-w,_{t\theta})+Q_{xz}w,_t\}]_0^l\ d\theta \tag{34}$$

Applying the related boundary control laws results in

$$\dot{V}=-[\int_0^{2\pi}\{u,_t^2+v,_t^2+w,_t^2+w,_{xt}^2+(v,_t-w,_{t\theta})^2\}]_{x=l}\ d\theta \tag{35}$$

For the second part of the Lemma, we utilize the following relation.

$$(I-A)X=X_0,\ \ X_0\in\mathbf{H} \tag{36}$$

It can be seen that

$$\begin{aligned}<(I-A)X,X>_{\mathbf{H}}&=\|X\|_{\mathbf{H}}^2+k_f\|x_4\|_{L^2(\Gamma_1)}^2+k_f\|x_6\|_{L^2(\Gamma_1)}^2+k_f\|x_8\|_{L^2(\Gamma_1)}^2\\&+k_m\|\partial x_6/\partial x\|_{L^2(\Gamma_1)}^2+k_m\|x_6-\partial x_8/\partial\theta\|_{L^2(\Gamma_1)}^2\geq\|X\|_{\mathbf{H}}^2\end{aligned} \tag{37}$$

where $\|X\|_{\mathbf{H}}^2=<X,X>$.

Using Lax–Milgram Lemma, one can easily prove that the above equation has a unique weak solution, [11; 10]. In particular, one can infer that:
$\Re(I-A)=H_{\Omega_3}^1(\Theta)\times L^2(\Theta)\times[H_{\Gamma_0}^2(\Omega)\times L^2(\Omega)]^3$ ($\Re(I-A)$ stands for the range of operator $(I-A)$).

On the other hand, it is clear that $D(A)$ is dense in $H^2(\Theta)\times L^2(\Theta)\times[H^4(\Omega)\times L^2(\Omega)]^3$, hence, according to Lumer-Phillips theorem, it is proved that A generates a C_0-semigroup of contractions [12]. Finally, one can obtain the following result:

$$\|X_0\|_{\mathbf{H}}\geq\|X\|_{\mathbf{H}} \tag{38}$$

Using Sobolev embedding theorem (Rellich-Kondrachov compact embedding theorem), [13], since $(I-A)^{-1}V$ is compactly embedded in $L^2(\Theta)\times L^2(\Theta)\times[L^2(\Omega)\times L^2(\Omega)]^3$, the compactness of the above-mentioned resolvent is evident.

Theorem 1. Let in the system (Eq. (14)), the initial condition X_0 belong to $D(A)$. Then, the system of Eq. (14) is well-posed [13].
Proof. Based on Lemma 1, it is evident that the system (Eq.(14)) is well-posed [12]. In addition, its strong solution has the following regularity [14]:

$$\begin{aligned}&\phi(t)\in C^0([0,t],H^2(\Theta)\cap H_{\Omega_3}^1(\Theta))\cap C^1([0,t],H_{\Omega_3}^1(\Theta))\cap C^2([0,t],L^2(\Theta))\\&w(t)\in C^0([0,t],H^4(\Omega)\cap H_{\Gamma_0}^2(\Omega))\cap C^1([0,t],H_{\Gamma_0}^2(\Omega))\cap C^2([0,t],L^2(\Theta))\end{aligned} \tag{39}$$

Now, we turn our attention to the proof of the asymptotic stability of the closed loop system.

Theorem 2. Using the boundary feedback control laws (Eq. (40)), the states of the system, X, will eventually tend toward zero.

$$U_i=-k_f\,x_{2(i+1)},\quad U_4=k_m\,\partial x_8/\partial x,\quad U_5=-k_mR(x_6-\partial x_8/\partial\theta)\qquad i=1,2,3 \tag{40}$$

Proof. The mechanical energy of the system as discussed previously, is

$$V(t)=<X,X> \tag{41}$$

By performing some algebraic operations and using Green's Lemma, the following can be obtained:

$$\dot{V}(t) = -\{k_f \|x_4\|^2_{L^2(\Gamma_1)} + k_f \|x_6\|^2_{L^2(\Gamma_1)} + k_f \|x_8\|^2_{L^2(\Gamma_1)} + k_m \|\partial x_6/\partial x\|^2_{L^2(\Gamma_1)} + k_m \|x_6 - \partial x_8/\partial\theta\|^2_{L^2(\Gamma_1)}\} \le 0 \tag{42}$$

where V is the total energy of the system and consists of two parts as follows:

$$V = V_{sh} + V_f \tag{43}$$

At this step, because of the compactness of resolvent $(I - A)^{-1}$, one can use LaSalle's invariant set theorem and therefore, it is sufficient to show that the following system has the trivial solution as its unique solution.

$$\left\{\begin{array}{l} \dot{X} = AX \quad \text{in} \quad \Omega \\ x_i = 0 \ \text{ and } \ M_{xx} = M_{x\theta} = Q_{xz} = 0, \ i = 3,..,8 \quad \text{in} \quad \Gamma_1 \\ \partial x_i/\partial z = 0, \ i = 3,..,8 \quad \text{in} \quad \Gamma_0 \\ \rho_0 \phi,_t = -p \quad \text{in} \quad \Omega \\ \rho_0 \phi,_{tt} + \rho_0 g \phi,_n = 0 \quad \text{in} \quad \Omega_2 \\ \phi = 0 \quad \text{in} \quad \Omega_3 \\ X(0) = X_0 \end{array}\right\} \tag{44}$$

Using the Holmgren uniqueness theorem, [14], one can easily show that the above system of equations admits only trivial solution. So, by regarding the LaSalle's invariant set theorem,

$$\lim_{t\to\infty} V(t) = 0 \tag{45}$$

which yields the desired stability.

It should be noted that when the structure is in the rest state, the fluid must have no motion. Because any motion in the fluid domain affects the structure and induces vibrations.

4. SIMULATION RESULTS

In this section, simulation results will be presented. Simulations are based on the finite element method. For the shell part, a 4 node, quadrilateral element with reduced integration has been used. This element is able to consider shear deformation or classical theory. This element utilizes uniform reduced integration to prevent shear and membrane locking. The element has several hourglass modes that may propagate over the mesh.

For the fluid part, we take advantage of an 8 node acoustic brick element. This element is capable of modeling wave propagation. The accuracy of the structural response is dependent on the accuracy of the fluid mesh.

a) Geometric properties of the shell containing fluid

The geometric model that has been considered is a circular cylinder with radius 0.5 *m* and height 0.5 *m* filled with water (l = 0.45 *m*). The shell thickness is 0.002 *m*.

The geometric view is as follows in Fig. 3.

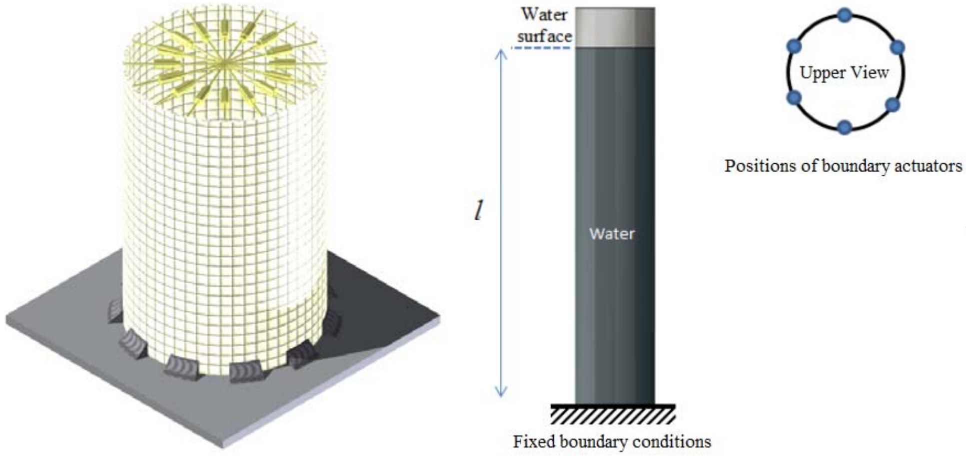

Fig. 3. Schematic view of shell containing fluid

b) Mechanical properties of the fluid-structure problem

The shell consists of seven layers with the following materials. The shell materials properties are as follows:

Table 1. Material Properties of the Shell

	E (GPa)	ν	Density (Kg/m^3)
Steel	200	0.3	7800

and the water properties are listed below

Table 2. Fluid (Water) Properties

Water	
Density (kg/m^3)	Bulk Modulus (GPa)
1000	2.2

The configuration of layer is as follows

Table 3. The Layers arrangement

Layer No.	Material	Thickness (mm)
Layers 1, 3, 5 and 7	Glass Epoxy Kind 1	0.35
Layers 2, 4 and 6	Glass Epoxy Kind 2	0.2

Layer 1.	Glass Epoxy Kind 1
Layer 2.	Glass Epoxy Kind 2
Layer 3.	Glass Epoxy Kind 1
Layer 4.	Glass Epoxy Kind 2
Layer 5.	Glass Epoxy Kind 1
Layer 6.	Glass Epoxy Kind 2
Layer 7.	Glass Epoxy Kind 1

Fig. 4. The Layers arrangement

c) Results

The simulation results consisting of controlled and uncontrolled shell vibrations are presented below. These results belong to a sample point at the upper edge of cylinder. These results show the effect of the boundary controllers.

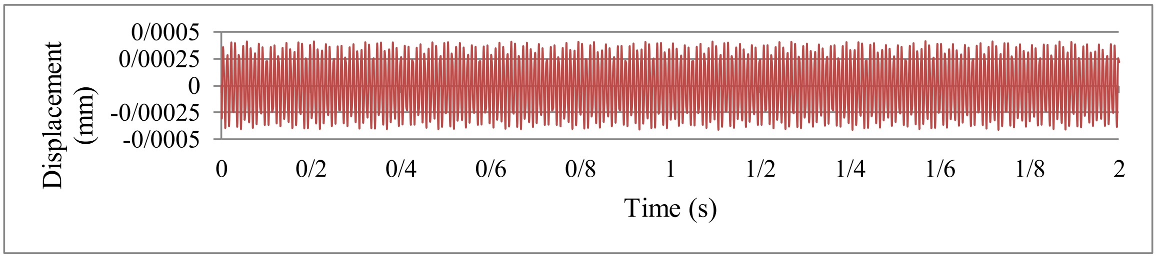

Fig. 5. Displacement of a sample point at upper edge of cylinder with $k_f = 0$ and $k_m = 0$

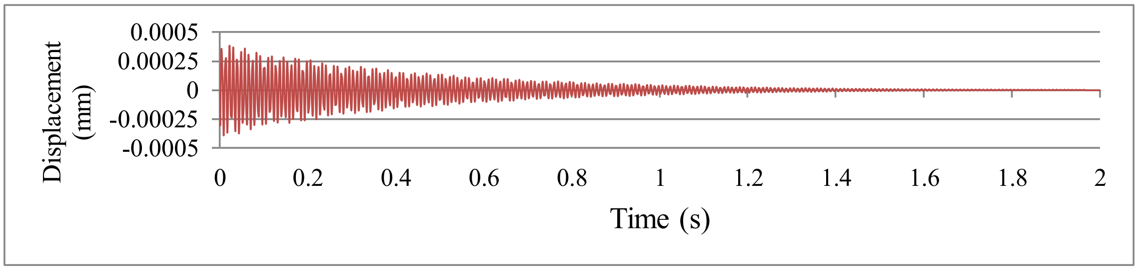

Fig. 6. Displacement of a sample point at upper edge of cylinder with $k_f = 1$ and $k_m = 1$

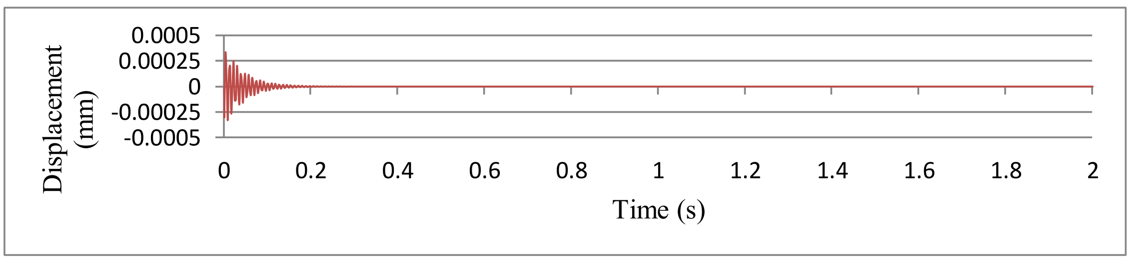

Fig. 7. Displacement of a sample point at upper edge of cylinder with $k_f = 10$ and $k_m = 10$

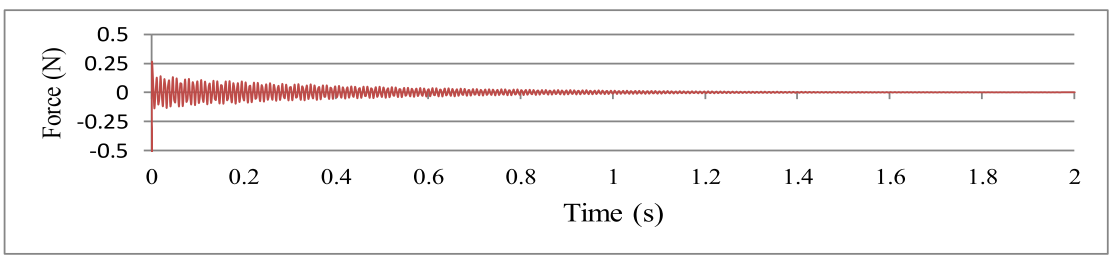

Fig. 8. Controller force with $k_f = 1$ and $k_\mathrm{m} = 1$

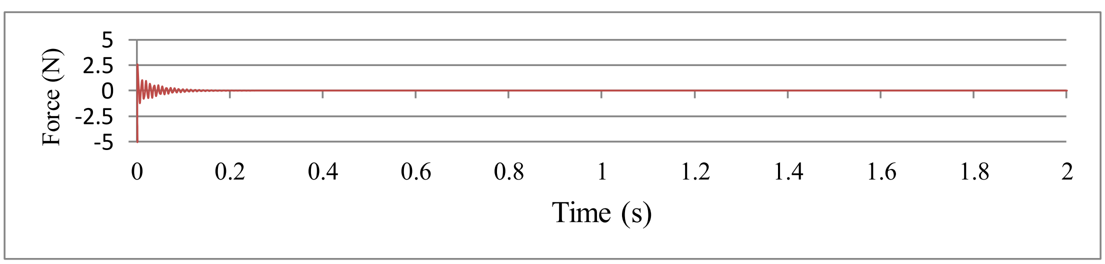

Fig. 9. Controller force with $k_\mathrm{f} = 1$ and $k_m = 1$

5. CONCLUSION

Utilizing boundary control method, Vibration suppression of a coupled fluid-structure problem has been addressed. The fluid has been assumed to be ideal and compressible having free surface. Finite element method (FEM) was utilized to perform the numerical simulation. In this method without installing any sensors and actuators in the fluid or structure domain, the fluid oscillation like motion and structure vibration has been stabilized. In fact, using simple feedbacks including boundary forces and moments, the asymptotic stabilization of the shell vibrations has been attained.

REFERENCES

1. Chapman, C. & Sorokin, S. (2005). The forced vibration of an elastic plate under significant fluid loading. sound and vibration, Vol. 281, pp. 719-741.
2. Amabili, M. (2003). Theory and experiments for large-amplitude vibrations of empty and fluid-filled circular cylindrical shells with imperfections. *Sound and Vibration*, Vol. 262, pp. 921-975.
3. Amabili, M., Pellicano, F. & Paidoussis, M. (2000). Nonlinear dynamics and stability of cicular cylindrical shells containing flowing fluid. part III: truncation effect without flow and experiments. *Sound and Vibration*, Vol. 237, pp. 617-640.
4. Morand, H. & Ohayon, R. (2001). *Fuid Structure Interaction*. John Wiley and Sons, New York.
5. Avalos, G., Lasiecka, I. & Rebarber, R. (2001). Well-posedness of a structural acoustics control model with point observation of the pressure. *Journal of Differential Equations*, Vol. 173, pp. 40-78.
6. Najafi, A., Eghtesad, M. & Daneshmand, F. (2010). Asymptotic stabilization of vibrating composite plates. *Systems & Control Letters*, Vol. 59, pp. 530-535.
7. Amabili, M. & Paidoussis, M. (2003). Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction. *Applied Mechanics Review*, Vol. 56, pp. 349-381.
8. Najafi, A. & Eghtesad, M. (2013). Asymptotic stabilization of composite Plates under fluid loading. *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, Vol. 37, 53-62.
9. Daneshmand, F. (2000). *Fluid-Structure interaction problems and its application in dynamic analysis of radial gates*. Shiraz University, Iran.
10. Rao, S. S. (2007). *Vibration of continuous systems*. Wiley.
11. Lions, J. (1971). *Optimal control of systems governed by partial differential equations*. Springer-Verlag.
12. Robinson, J. (2001). *Infinite-dimensional dynamical system*. Cambridge University Press, (2001).
13. Yosida, K. (1995). *Functional analysis*. Springer-Verlag.
14. Lions, J. & Magenes, E. (1973). *Non-homogeneous boundary value problems and applications*. Springer-Verlag, (1973).