

## SELF-TUNING OF AN INTERVAL TYPE-2 FUZZY PID CONTROLLER FOR A HEAT EXCHANGER SYSTEM\*

H. BEIRAMI<sup>1\*\*</sup> AND M. M. ZERAFAT<sup>2</sup>

<sup>1</sup> Dept. of Instrumentation and Automation Engineering, Shiraz University, Shiraz, I. R. of Iran  
Email: Beirami.hamed@yahoo.com

<sup>2</sup> Faculty of Advanced Technologies, Nano Chemical Engineering Dept., Shiraz University, Shiraz, I. R. of Iran

**Abstract**– A novel technique is investigated for PID controller adaptation in order to control the temperature of a liquid-phase reactor tank by using a heat exchanger system. As for nonlinearity, time delay problems and model uncertainties introduced by the heat exchanger, an interval type-2 fuzzy system (IT2FS) is implemented to enhance and improve the total control performance. Moreover, the fuzzy inference rules which enable the adaptive adjustment of PID parameters are established based on error and error variations. Target tracking, oscillation control and error evaluation for the proposed controller are compared with previously performed control strategies on the mentioned heat exchanger system. The results show that the adaptive technique for PID gain based on IT2FS has lower error and strengthened capacity for external oscillation control and also an acceptable tracking capability.

**Keywords**– Fuzzy PID controller, self-tuning control, interval type-2 fuzzy, heat exchanger system

### 1. INTRODUCTION

The dynamics of many industrial systems including heat exchangers is accompanied by complexity, time delay, nonlinearity and uncertainty. Such systems are usually controlled by fixed-parameter linear controller structures with effective performance around the design conditions. As a result, control systems have to be adapted to the new conditions confronted due to variations throughout the system life-time and controller gains are also required to be adjusted to various possible operating conditions. Therefore, online adjustment of controller parameters is required to maintain the desired performance of the system as a whole. Due to simplicity and robustness, PID controllers are the most commonly used control algorithms in industrial systems. In fact, PID controllers and their derivatives are used in ~ 90% of industrial processes [1, 2]. Widespread and pervasive application of PID controllers has led to the development of various PID tuning and adaptation techniques [2]. However, the search is still on for new tuning and adaptation techniques [3]. In the current article, a novel method is investigated to control the temperature of a heat exchanger, in order to adjust the temperature of the flowing liquid to decrease the time needed to gain the desired temperature and also to make the system more robust in terms of disturbances.

Heat exchangers are commonly used in industrial applications for heat exchange between different stages within a plant. Heat exchange control is a complex process due to nonlinear behaviour and complexity as a result of phenomena, like leakage, friction, temperature dependent flow properties, contact resistance, unknown fluid properties, etc. [4, 5]. However, such plants are already controlled properly by conventional PID controllers (or its derivatives) [4-6].

Many conventional process control strategies are performed on heat exchange systems. PID [7-9],

---

\*Received by the editors January 14, 2014; Accepted May 25, 2014.

\*\*Corresponding author

IMC-PID [10], GPC [11, 12] and fuzzy [13] control strategies are performed and compared to MPC. In this study, Type-2 fuzzy system is applied for the optimization of PID controller and parameter tuning. A combination of fuzzy type-2 and conventional PID enables us to utilize the advantages of both systems at the same time.

Fuzzy logic was introduced by Zadeh [14, 15] as a model of human thinking process in order to remove the gap between the precision of mathematics and the innate imprecision of the real world. Fuzzy thinking provides a systematic procedure for transforming a knowledge base into a nonlinear mapping. Over past two decades, it has been shown that fuzzy systems can be considered as universal approximators; hence approximating the continuous functions on a compact set to a given accuracy [16-20]. The main property distinguishing fuzzy logic is the capacity to represent and model the imprecision and uncertainty by attributing a value at the interval  $[0, 1]$  to each point in a fuzzy set. However, the imprecision in such a classical fuzzy system - sometimes called type-1 fuzzy logic system (T1FLS) - is not fully exploited and can bring about unsatisfactory performance.

Over the past few years, considerable attention has been devoted to another fuzzy system called type-2 FLS (T2FLS). In T2FLS, the uncertainty is represented using a function, which is a type-1 fuzzy number itself. The functions of type-2 fuzzy sets have a 3D membership pattern and include a footprint of uncertainty (FOU) with the new 3<sup>rd</sup> dimension of type-2 fuzzy sets. A FOU provides additional degrees of freedom that make it possible to model and handle uncertainties directly. Consequently, T2FLS has the potential to outperform the T1FLS in such cases [21, 22]. Such an advantage is quite important, keeping in mind that modelling the linguistic information and decision making are the main applications of FLS [23]. A comparison of T2FLS and T1FLS is also given in [24].

Mamdani recognized the feasibility of fuzzy logic concept for controlling dynamic systems [25]. Mamdani and Assilian [26] developed the first fuzzy logic controller (FLC). Interval type-2 fuzzy logic systems have been successfully implemented for controller design. The advantageous view of type-2 fuzzy controller has been demonstrated in several applications such as controlling the liquid-level [27, 28], multi-machine power systems voltage [29], autonomous mobile robots [30, 31] and nonlinear dynamic plants [32] control. Furthermore, type-2 fuzzy model based on model predictive control structures is also suggested [33]. Moreover, state observers based on indirect adaptive internal type-2 fuzzy controller are introduced [34]. In addition, control structures based on ordinary fuzzy logic systems have been generalized to type-2 fuzzy systems [35, 36].

The main shortcoming of fuzzy control strategy is the uncertain knowledge used for building the fuzzy rules, resulting in uncertain antecedents or consequents and thus uncertain antecedent or consequent membership functions [37] and complexity and time-consuming computational time at the same time. Type-1 fuzzy systems are unable to handle such uncertainty. On the other hand, type-2 fuzzy systems are very useful in the determination of exact membership function.

Many research efforts have been dedicated to the design of fuzzy-PID type controllers and its derivatives. Automate control of rotary dental instrument files fail through the development of a fuzzy logic controller to maintain the file [38]. Robust fuzzy PID control schemes are proposed by incorporating an optimal fuzzy reasoning into a well-developed PID type of control framework [39]. Also, bounded-input bounded-output (BIBO) stability analysis of a fuzzy PID control system has been performed [40]. A function-based evaluation approach is proposed for a systematic study of F-PID like controllers addressing simplicity and nonlinearity issues [41]. The optimal PID controller parameters for the adaptive Particle Swarm Optimization method based on Cloud Theory are also applied to fuzzy PID controllers [42].

## 2. HEAT EXCHANGER SYSTEMS

The heat exchanger system used in this study is investigated in other studies for control purposes and identified by experimental data in the reference [43]. A typical chemical reactor with the accompanying heating system is shown in Fig. 1. Vapour flow into the heating system is adjusted and manipulated by a control valve in order to change the liquid temperature inside the tank at the desired set-point. Temperature variations of the inflow to the main tank can be considered as the main disturbance in the whole system [13].

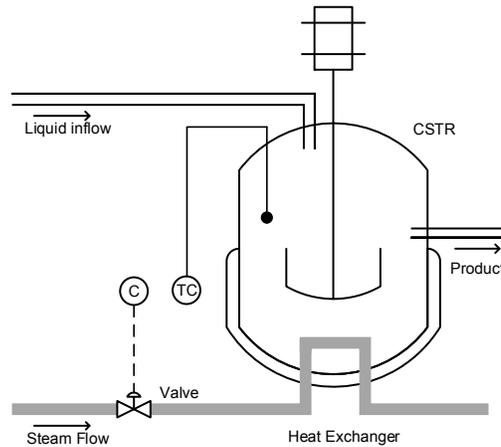


Fig. 1. Reactor with a heat exchanger system

A step change is applied to the the control valve voltage and the influence on temperature is recorded as a function of time in order to obtain the first order model. The normalized measured response is also illustrated in Fig. 2. This type of response is a schematic of first-order-plus-dead time (FOPDT) systems. The two-point method is also used to identify the model function and its parameters [44]. This technique is based on the calculation of  $t_1$  and  $t_2$  points which are 28.3 % and 63.2% fractions of the system final response time, respectively. These are used for the estimation of time constant,  $\tau = 1.5(t_2 - t_1)$  and dead time,  $\theta = t_2 - \tau$ . The heat exchanger transfer function is calculated as follows by taking  $t_1 = 21.8$  and,  $t_2 = 36$ .

$$H(s) = \frac{e^{-\theta s}}{\tau s + 1} \tag{1}$$

Here,  $\theta = 14.7$  sec and  $\tau = 21.3$  sec.

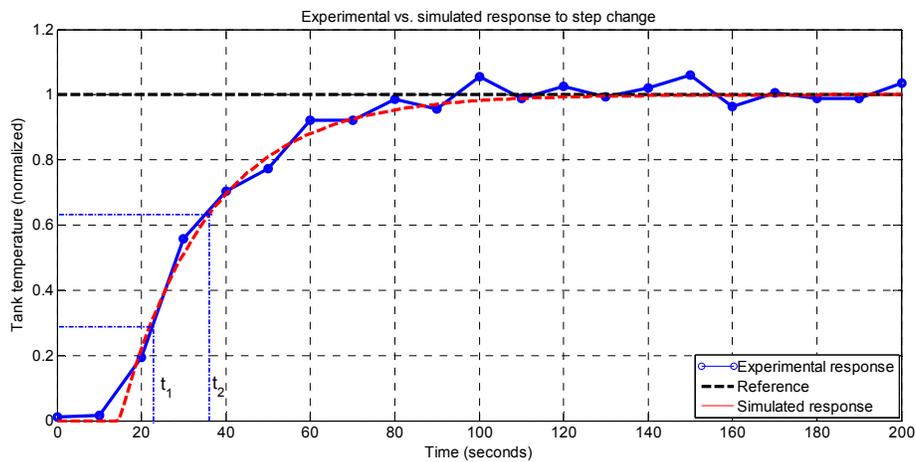


Fig. 2. The normalized measured step response of the plant

### 3. INTERVAL TYPE-2 FUZZY SETS

A type-2 fuzzy set in the universal set  $X$  is denoted as  $\tilde{A}$ , which is characterized by a type-2 membership function (MF)  $\mu_{\tilde{A}}(x)$  in Eq. (2) and referred to as a secondary membership function or a secondary set, which is a type-1 fuzzy set at the  $[0,1]$  interval.  $f_x(u)$  is a secondary grade, which is the amplitude of a secondary membership function; i.e,  $0 \leq f_x(u) \leq 1$ . The domain of a secondary membership function is called the primary membership of  $x$ .  $J_x$  is the primary membership of  $x$ , where  $u \in J_x \subseteq [0,1] \forall x \in X$ ;  $u$  is a fuzzy set at the  $[0,1]$  interval, rather than a crisp point [45]:

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x)/x = \int_{x \in X} \left[ \int_{\mu \in J_x} f_x(\mu)/\mu \right] /x, J_x \subseteq [0,1] \tag{2}$$

when  $f_x(u) = 1, \forall u \in J_x \subseteq [0,1]$ . Secondary MFs are interval sets such that  $\mu_{\tilde{A}}(x)$  can be called an interval type-2 MF [45-48]. Therefore, the type-2 fuzzy set can be rewritten as:

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x)/x = \int_{x \in X} \left[ \int_{\mu \in J_x} 1/\mu \right] /x, J_x \subseteq [0,1] \tag{3}$$

Also, a Gaussian primary MF with an uncertain mean and fixed standard deviation having an interval type-2 secondary MF can be called an interval type-2 Gaussian MF. A 2D interval type-2 Gaussian MF with an uncertain mean at the  $[m_1, m_2]$  interval and a fixed standard deviation  $\sigma$  is shown in Fig. 3, which can be expressed as:

$$\mu_{\tilde{A}}(x) = \exp \left[ -\frac{1}{2} \left( \frac{x - m}{\sigma} \right)^2 \right], \quad m \in [m_1, m_2] \tag{4}$$

It is obvious that the type-2 fuzzy set is described within a region called the footprint of uncertainty (FOU) which is bounded by upper and lower MFs denoted by  $\bar{\mu}_{\tilde{A}}(x)$  and  $\underline{\mu}_{\tilde{A}}(x)$ , respectively.

In this paper, the input and output variables will be represented by IT2FSs as they are simpler to be used in comparison with the general T2FSs and also distribute the uncertainty evenly among all admissible primary memberships [48].

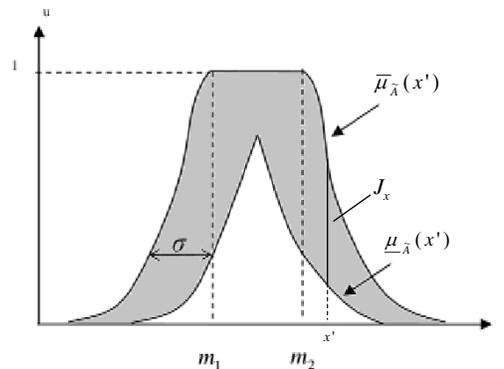


Fig. 3. Interval type-2 fuzzy set with an uncertain mean at the  $[m_1, m_2]$  interval

### 4. INTERVAL TYPE-2 FUZZY LOGIC CONTROLLER

The basics of fuzzy logic are the same for  $T_1$  and  $T_2$  sets and are also similar for any type- $n$  set in general. Higher versions just indicate a higher ‘‘degree of fuzziness’’. Since the nature of membership functions change by  $n$  value, the operation is dependent on the membership functions. However, the basics of fuzzy logic are independent of the nature of membership functions and hence remain unchanged [24].

IT2FLC contains four fuzzifier components: inference engine, rule base, and output processing that is

inter-connected as shown in Fig. 4 [49]. In IT2FLC, a crisp input is first fuzzified into input IT2FSs, which activate the inference engine and the rule base to produce the output IT2FSs [50]. The IT2FLC rules will remain the same as T1FLC but the antecedents and/or the consequent will be represented by IT2FSs. The IT2 fuzzy outputs of the inference engine are then processed by the type reducer, which combines the output sets and performs a centroid calculation that leads to T1FSs, called type-reduced sets. After the type-reduction process, the type-reduced sets are defuzzified to obtain crisp outputs by averaging the type-reduced set.

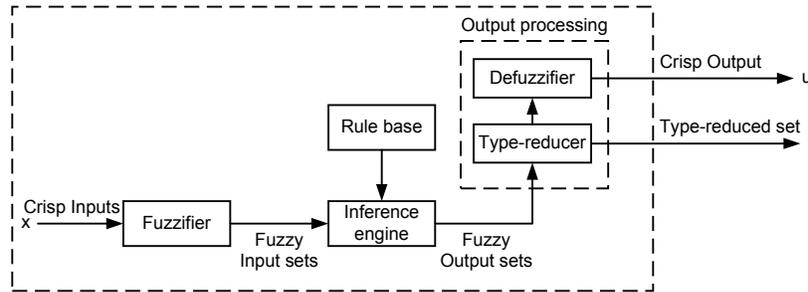


Fig. 4. Structure of the type-2 fuzzy logic system

Consider a type-2 FLS having  $p$  inputs  $x_1 \in X_1, \dots, x_p \in X_p$  and one output  $y \in Y$ . Type-2 fuzzy rule base consists of a collection of IF-THEN rules, as in type-1.  $M$  rules are assumed and the rule of a type-2 relation between the input space  $X_1 \times X_2 \times \dots \times X_p$  and output space  $Y$  can be expressed as:

$$R^l : IF x_1 \text{ is } \tilde{F}_1^l \text{ and } \dots \text{ and } x_p \text{ is } \tilde{F}_p^l, \quad THEN y \text{ is } \tilde{G}^l \quad l = 1, 2, \dots, M \quad (5)$$

where  $\tilde{F}_j^l$ s are antecedent type-2 sets ( $j = 1, 2, \dots, p$ ) and  $\tilde{G}^l$ s are consequent type-2 sets.

The inference engine combines the rules and gives a mapping from the input to the output type-2 fuzzy sets. Thus, we have to compute the unions and intersections of type-2 sets as well as the compositions of type-2 relations. The output of the inference engine block is a type-2 set. By using the extension principle of the type-1 defuzzification method, type reduction transforms type-2 output sets of the FLS to a type-1 set called ‘‘reduced-type set’’. This set may then be defuzzified to obtain a single crisp value. Many type reduction procedures such as centroid, height, modified weight, and center-of-sets are available [45, 47-48, 51], among which the center-of-sets used in this paper is described as follows:

$$Y_{cos}(Y^1, \dots, Y^M, F^1, \dots, F^M) = [y_l, y_r] = \int_{y^1} \dots \int_{y^M} \int_{f^1} \dots \int_{f^M} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \quad (6)$$

where  $Y_{cos}$  is the interval set determined by two end points  $y_l$  and  $y_r$  and  $f^i \in F^i = [f^i, \bar{f}^i]$ . In the meantime, an interval type-2 FLS with singleton fuzzification and meet under minimum t-norm  $\underline{f}^i$  and  $\bar{f}^i$  can be obtained as:

$$\underline{f}^i = \min [\underline{\mu}_{\tilde{F}_1^i}(x_1) \dots \underline{\mu}_{\tilde{F}_p^i}(x_p)] \quad (7)$$

and,

$$\bar{f}^i = \min [\bar{\mu}_{\tilde{F}_1^i}(x_1) \dots \bar{\mu}_{\tilde{F}_p^i}(x_p)] \quad (8)$$

Also,  $y^i \in Y^i$  and  $Y^i = [y_l^i, y_r^i]$  is the centroid of the type-2 interval consequent set  $\tilde{G}^l$  (the centroid of type-2 fuzzy set) [1-3].  $y$  ( $y \in Y_{cos}$ ) can also be expressed as:

$$y = \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \quad (9)$$

where  $y$  is a monotonic increasing function of  $y^i$ . Also,  $y_l$  (Eq. 6) is the minimum associated only with  $y_r^i$ , and  $y_r$  (Eq. 6) is the maximum associated only with  $y_r^i$ . Note that  $y_l$  and  $y_r$  depend on a mixture of  $\underline{f}^i$  or  $\bar{f}^i$  values. Hence, the left-most and the right-most points ( $y_l$  and  $y_r$ , respectively) can be expressed as [47]:

$$y_l = \frac{\sum_{i=1}^M f_l^i y_l^i}{\sum_{i=1}^M f_l^i} \quad (10)$$

and,

$$y_r = \frac{\sum_{i=1}^M f_r^i y_r^i}{\sum_{i=1}^M f_r^i} \quad (11)$$

The Karnik-Mendel (KM) algorithm is briefly introduced in the following paragraphs [52]. Without loss of generality, assume that  $y_l^i$  and  $y_r^i$  are arranged in an ascending order ( $y_l^1 \leq y_l^2 \leq \dots \leq y_l^M$  and  $y_r^1 \leq y_r^2 \leq \dots \leq y_r^M$ ).  $y_l$  is then computed following these steps:

**Step1.** Eq. 12 is solved by initially setting  $f_l^i = (\bar{f}^i + \underline{f}^i)/2$  for  $i = 1, 2, \dots, M$ , where  $\bar{f}^i$  and  $\underline{f}^i$  have been pre-computed using (7), (8) and letting  $y_l^i = y_l$ .

**Step2.** Find  $L$  ( $1 \leq L \leq M - 1$ ) so that  $y_l^L \leq y_l^i \leq \dots \leq y_l^{L+1}$ .

**Step3.** Compute  $y_r$  in Eq. 12 with  $f_l^i = \bar{f}^i$  for  $i \leq L$  and  $f_r^i = \underline{f}^i$  for  $i > L$  and letting  $y_r^i = y_l$ .

**Step4.** If  $y_r^i \neq y_l^i$ , then go to Step 5. If  $y_r^i = y_l^i$ , then set  $y_l = y_r^i$  and go to Step 6.

**Step5.** Let  $y_l^i = y_r^i$  and return to Step 2.

The separation point ( $L$ ) can be determined using this algorithm, one side using lower firing strengths ( $\underline{f}^i$ ) and the other upper firing strengths ( $\bar{f}^i$ ). Therefore,  $y_l$  can be given as:

$$y_l = \frac{\sum_{i=1}^L \underline{f}^i y_l^i + \sum_{i=L+1}^M \bar{f}^i y_l^i}{\sum_{i=1}^L \underline{f}^i + \sum_{i=L+1}^M \bar{f}^i} = \sum_{i=1}^L \underline{q}_l^i y_l^i + \sum_{i=R+1}^M \bar{q}_l^i y_l^i \quad (12)$$

Where  $\underline{q}_l^i = \underline{f}^i / D_l$ ,  $\bar{q}_l^i = \bar{f}^i / D_l$  and  $D_l = \sum_{i=1}^L \underline{f}^i + \sum_{i=L+1}^M \bar{f}^i$ .

$y_r$  is also computed based on a similar procedure. In Step 2,  $R$  ( $1 \leq R \leq M - 1$ ) is to be determined, so that  $y_r^R \leq y_r^i \leq \dots \leq y_r^{R+1}$ . In Step 3,  $f_r^i = \underline{f}^i$  for  $i \leq R$  and  $f_r^i = \bar{f}^i$  for  $i > R$ .  $y_r$  can also be rewritten as:

$$y_r = \frac{\sum_{i=1}^R \underline{f}^i y_r^i + \sum_{i=R+1}^M \bar{f}^i y_r^i}{\sum_{i=1}^R \underline{f}^i + \sum_{i=R+1}^M \bar{f}^i} = \sum_{i=1}^R \underline{q}_r^i y_r^i + \sum_{i=R+1}^M \bar{q}_r^i y_r^i \quad (13)$$

Where  $\underline{q}_r^i = \underline{f}^i / D_r$ ,  $\bar{q}_r^i = \bar{f}^i / D_r$  and  $D_r = \sum_{i=1}^R \underline{f}^i + \sum_{i=R+1}^M \bar{f}^i$ .

The defuzzified crisp output from an IT2FLS is the average of  $y_r$  and  $y_l$ .

$$y(x) = \frac{y_l + y_r}{2} \quad (14)$$

## 5. ADAPTIVE FUZZY PID CONTROLLER DESIGN USING INTERVAL TYPE-2 FUZZY LOGIC SYSTEM

Adaptive interval type-2 fuzzy PID (AIT2FPID) control based on a PID algorithm performs the reasoning through calculating the error ( $e$ ) and error derivative ( $ec$ ) of the system by using type-2 fuzzy inference rules and adjusts the PID parameters by fuzzy matrix rule tables. In designing adaptive fuzzy controllers, the error and its derivative are assumed as inputs, which can satisfy the need for self-tuning of PID parameters based on various ( $e$ ) and ( $ec$ ) values at different times. The PID algorithm is also presented as:

$$\begin{aligned}
 u(t) &= [k_p(t)e(t)] + \int_0^t [k_i(\tau)e(\tau)]d\tau + \frac{d[k_D(t)e(t)]}{dt} = [1 \quad 1 \quad 1] \begin{bmatrix} [k_p(t)e(t)] \\ \int_0^t [k_i(\tau)e(\tau)]d\tau \\ \frac{d[k_D(t)e(t)]}{dt} \end{bmatrix} \\
 &= \psi \cdot \Gamma(k_p(t)e(t), k_i(\tau)e(\tau), k_D(t)e(t))
 \end{aligned} \tag{15}$$

where,

$$\left\{ \begin{array}{l} \psi = [1 \quad 1 \quad 1] \\ \Gamma = \left[ \begin{array}{l} k_p(t)e(t) \quad \int_0^t [k_i(\tau)e(\tau)]d\tau \quad \frac{d[k_D(t)e(t)]}{dt} \end{array} \right] \end{array} \right. \tag{16}$$

The PID controller described by Eq. (15) is equivalent to:

$$\begin{aligned}
 u(t) &= [k_p(t)e(t)] + \int_0^t [k_i(\tau)e(\tau)]d\tau + \frac{d[k_D(t)e(t)]}{dt} \\
 &= [k_p^0 + \Delta k_p(t)]e(t) + \int_0^t [k_i^0 + \Delta k_i(\tau)]e(\tau)d\tau + \frac{d[k_D^0 + \Delta k_D(t)]e(t)}{dt}
 \end{aligned} \tag{17}$$

where,

$$\left\{ \begin{array}{l} k_p(t) = k_p^0 + \Delta k_p(t) \\ k_i(t) = k_i^0 + \Delta k_i(t) \\ k_D(t) = k_D^0 + \Delta k_D(t) \end{array} \right. \tag{18}$$

$k_p^0$ ,  $k_i^0$  and  $k_D^0$  are system parameters and these time invariant constants are formerly adjusted for the PID controller. On the other hand,  $\Delta k_p(t)$ ,  $\Delta k_i(t)$  and  $\Delta k_D(t)$  are time-varying parameters which can be adapted according to practical situations in real-time experiments. Therefore, it may be inferred that the adaptation of  $\Delta k_p(t)$ ,  $\Delta k_i(t)$  and  $\Delta k_D(t)$  leads to the adaptation of  $k_p(t)$ ,  $k_i(t)$  and  $k_D(t)$ .

In order to expand the adaptation of  $\Delta k_p(t)$ ,  $\Delta k_i(t)$  and  $\Delta k_D(t)$ , Eq. (11) can be modified as follows:

$$\begin{aligned}
 u(t) &= k_p^0 e(t) + k_i^0 \int_0^t e(\tau) d\tau + k_D^0 \frac{de(t)}{dt} + \Delta k_p(t)e(t) + \int_0^t \Delta k_i(\tau) e(\tau) d\tau \\
 &\quad + \frac{d[\Delta k_D(t)e(t)]}{dt} = u^0(t) + \Delta u(t)
 \end{aligned} \tag{19}$$

where,

$$\left\{ \begin{array}{l} u^0(t) = k_p^0 e(t) + k_i^0 \int_0^t e(\tau) d\tau + k_D^0 \frac{de(t)}{dt} \\ \Delta u(t) = \Delta k_p(t)e(t) + \int_0^t \Delta k_i(\tau) e(\tau) d\tau + \frac{d[\Delta k_D(t)e(t)]}{dt} \end{array} \right. \tag{20}$$

Equation (19) shows that the proposed PID controller can be divided into the main and relay controllers connected in a parallel configuration. The main controller is specified through parameters  $k_p^0$ ,  $k_I^0$  and  $k_D^0$  and the corresponding output is assumed as  $u^0(t)$  here. The relay controller is distinguished through parameters  $\Delta k_p(t)$ ,  $\Delta k_I(t)$  and  $\Delta k_D(t)$ . Here, the output would be  $\Delta u(t)$ . The relay controller output can be equivalent to the following relation:

$$\Delta u(t) = \Delta e_p(t) + \int_0^t \Delta e_I(t) + \frac{d[\Delta e_D(t)]}{dt} \tag{21}$$

where,

$$\begin{cases} \Delta e_p(t) = \Delta k_p(t)e(t) \\ \Delta e_I(t) = \Delta k_I(t)e(t) \\ \Delta e_D(t) = \Delta k_D(t)e(t) \end{cases} \tag{22}$$

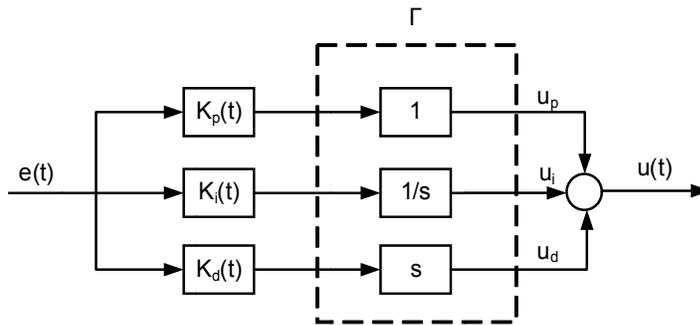


Fig. 5. The PID Controller structure

$\Delta e_p(t)$ ,  $\Delta e_I(t)$ ,  $\Delta e_D(t)$  are proportional to the system error  $e(t)$ . Variation of  $\Delta k_p(t)$ ,  $\Delta k_I(t)$  and  $\Delta k_D(t)$  can lead to changes in  $\Delta e_p(t)$ ,  $\Delta e_I(t)$  and  $\Delta e_D(t)$  and vice versa. As a result, if the relationship between these three signals and the system error is assumed to be any generalized function, the change of the three signals implies the implicit adaptation of  $\Delta k_p(t)$ ,  $\Delta k_I(t)$  and  $\Delta k_D(t)$ . So, the implicit adaptation approach is not to make the PID controller parameters adaptive but to produce a series of signals adaptively so that PID the parameters  $k_p(t)$ ,  $k_I(t)$  and  $k_D(t)$  become implicitly adaptive. These three signals are the outputs of type-2 fuzzy controller. Here, the error and the error derivative are considered as inputs (Fig. 5). The structure of the AIT2FPID controller is also illustrated in Fig. 6.

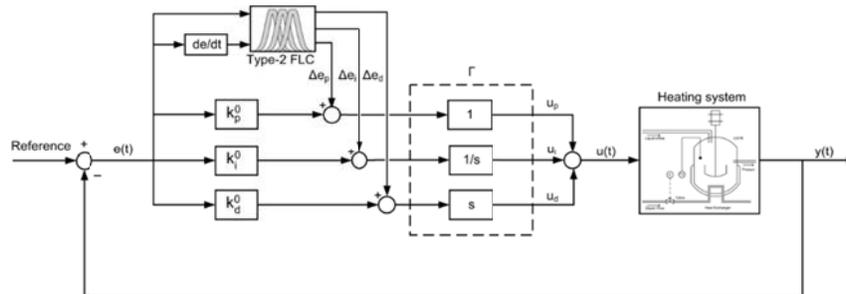


Fig. 6. The structure of adaptive type-2 fuzzy PID control system

In a fuzzy system, the type-2 fuzzy membership function is employed to convert the crisp data to a degree of membership by taking into account the uncertainties as well. All input/output fuzzy sets of the type-2 fuzzy controller consist of seven linguistic variables {NB, NM, NS, ZO, PS, PM, PB}.  $\mu_{\bar{A}}s$  are defined as the input membership functions of the type-2 fuzzy sets. These membership functions consist of system error and error derivative defined at the interval [-30, 30] of the universal set (Fig. 7).

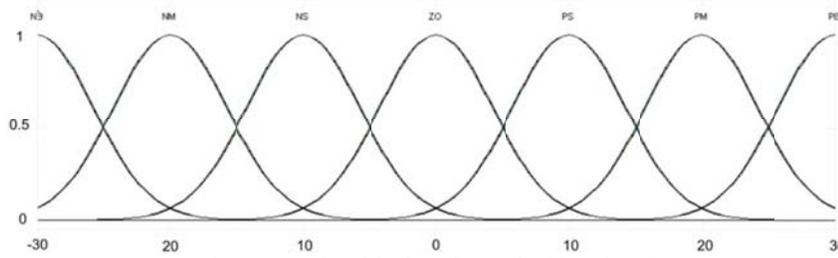


Fig. 7. Membership functions of  $e(t)$  and  $ec(t)$

Table 1. Control rules for  $\Delta e_p$

$\Delta e_I$		Error Change (ec)						
		NB	NM	NS	ZO	PS	PM	PB
Error (e)	NB	PB	PB	PM	PM	PS	ZO	ZO
	NM	PB	PB	PM	PS	PS	ZO	NS
	NS	PM	PM	PM	PS	ZO	NS	NS
	ZO	PM	PM	PS	ZO	NS	NM	NM
	PS	PS	PS	ZO	NS	NS	NM	NM
	PM	PS	ZO	NS	NM	NM	NM	NB
	PB	ZO	ZO	NM	NM	NM	NB	NB

Table 2. Control rules for  $\Delta e_I$

$\Delta e_I$		Error Change (ec)						
		NB	NM	NS	ZO	PS	PM	PB
Error (e)	NB	NB	NB	NM	NM	NS	ZO	ZO
	NM	NB	NB	NM	NS	NS	ZO	ZO
	NS	NB	NM	NS	NS	ZO	PS	PS
	ZO	NM	NM	NS	ZO	PS	PM	PM
	PS	NM	NS	ZO	PS	PS	PM	PB
	PM	ZO	ZO	PS	PS	PM	PB	PB
	PB	ZO	ZO	PS	PM	PM	PB	PB

Table 3. Control rules for  $\Delta e_D$

$\Delta e_D$		Error Change (ec)						
		NB	NM	NS	ZO	PS	PM	PB
Error (e)	NB	PS	NS	NB	NB	NB	NM	PS
	NM	PS	NS	NB	NM	NM	NS	ZO
	NS	ZO	NS	NM	NM	NS	NS	ZO
	ZO	ZO	NS	NS	NS	NS	NS	ZO
	PS	ZO	ZO	ZO	ZO	ZO	ZO	ZO
	PM	PB	NS	PS	PS	PS	PS	PB
	PB	PB	PM	PM	PM	PS	PS	PB

$\mu_{\tilde{A}}s$  are the output membership functions producing the adaptive signals  $\Delta e_p(t)$ ,  $\Delta e_I(t)$  and  $\Delta e_D(t)$  at  $[-1.5 \ 1.5]$ ,  $[-0.4 \ 0.4]$ ,  $[-7 \ 7]$  intervals of the universal set, respectively (Fig. 8).

According to [53], three rules of thumb can be used in tuning  $\Delta e_p$ ,  $\Delta e_I$  and  $\Delta e_D$  as follows:

- (1) If  $|e|$  is large, then  $\Delta e_p$  should be large and  $\Delta e_D$  should be small so that the system has a quick response. Meanwhile, the integral action should be limited (usually  $\Delta e_I=0$ ) lest the system experience large overshoots.
- (2) If  $|e|$  is moderate, then  $\Delta e_p$  should be small. The quantity of  $\Delta e_D$  is more important to obtain a small overshoot.

(3) If  $|e|$  is small, then  $\Delta e_P$  and  $\Delta e_I$  should be large to enable the system have a better steady-state performance. When  $|ec|$  is small,  $\Delta e_D$  should be large and reverse. In this way, oscillations near the set-point can be avoided.

The control strategy in the proposed AFPIDC can be expressed as: if  $e$  is (. . .) and  $ec$  is (. . .), then  $\Delta e_P$  is (. . .),  $\Delta e_I$  is (. . .) and  $\Delta e_D$  is (. . .). The fuzzy rules implemented for computing  $\Delta e_P(t)$ ,  $\Delta e_I(t)$ , and  $\Delta e_D(t)$  are shown in Tables 1, 2 and 3.

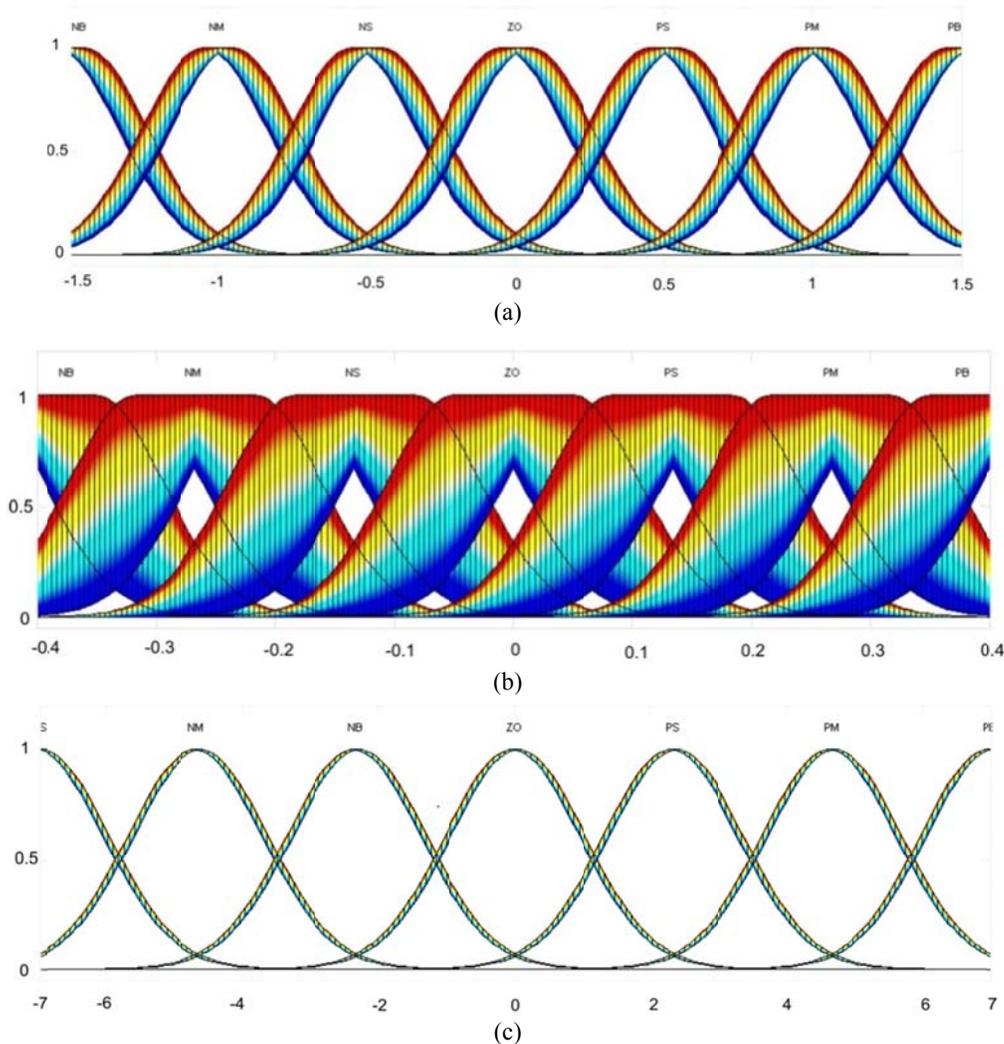


Fig. 8. Membership functions of output (a)  $\Delta e_P$  (b)  $\Delta e_I$  (c)  $\Delta e_D$

## 6. RESULTS AND DISCUSSION

A heating system is used to simulate the reservoir temperature adjustment procedure. The sampling rate is set to 0.1 in all simulations. Type 2 fuzzy system is used to tune the PID controller gains. Type 2 fuzzy controller outputs:  $\Delta k_P(t)$ ,  $\Delta k_I(t)$  and  $\Delta k_D(t)$ , are shown in Fig. 9 for tracking and disturbance rejection.

The performance of the suggested AIT2FPID controller is compared with adaptive type-1 fuzzy PID (AT1FPID), model predictive control (MPC), conventional FLC and conventional PID controllers for 600 s tracking. Besides, considering the temperature fluctuations in the reservoir inflow as the main disturbance, disturbance rejection is done in 200 s of operation. The conventional PID and main fuzzy PID controller parameters described in the previous section are assumed equal and presented in Table 4.

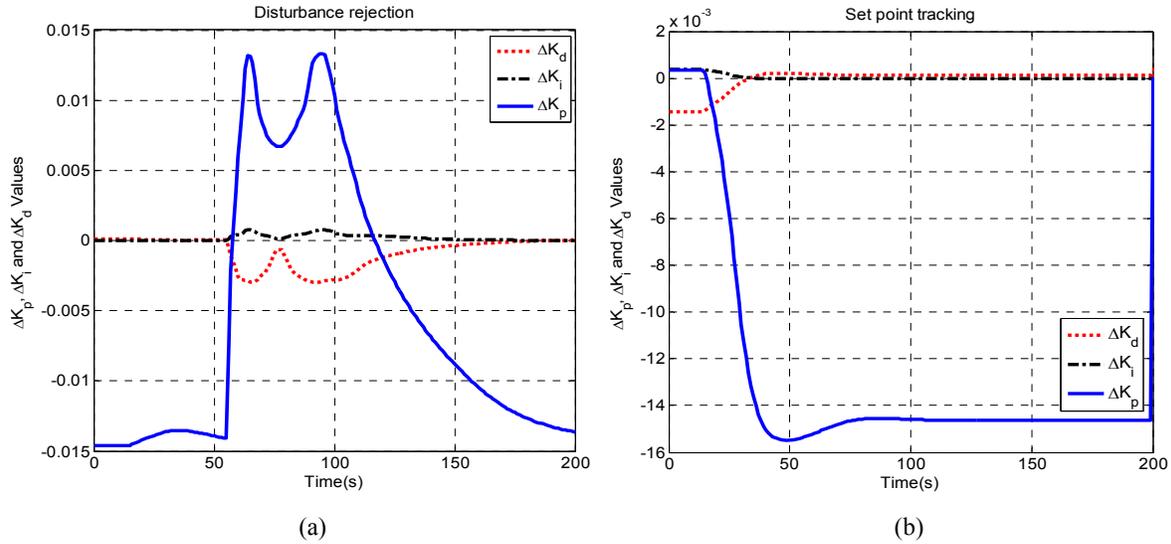


Fig. 9. The proposed controller output values (a) Disturbance rejection (b) Tracking

Table 4. PID parameters

Parameters	PID, AT1FPID and AIT2FPID
$k_p^0$	1.129
$k_i^0$	0.043
$k_d^0$	3.040

There are 4 evaluation methods for the closed-loop transient response of the proposed control systems and also to make a precise comparison. Two of these methods deal with classic performance measuring criteria such as “Maximum overshoot” (%OS) and “Settling time” (Ts) and two others are also described as:

(i) Integral absolute error (IAE):

$$IAE = \int_0^{\infty} |e(t)| dt \tag{23}$$

(ii) Integral time absolute error (ITAE):

$$ITAE = \int_0^{\infty} t|e(t)| dt \tag{24}$$

The tracking performance of AIT2FPID, AT1FPID and the conventional PID controllers are illustrated in Fig. 10. The performance comparison using IAE and ITAE criteria for 200 s are depicted in Fig. 11. As it is obvious, tracking accuracy and controller error have both led to better results in AIT2FPID in comparison with AT1FPID and the conventional PID considerably according to Table 5. Furthermore, %OS and Ts are remarkably lower in the former method.

Table 5. Performance of PID tuning techniques (Tracking 200 sec)

Performance Criteria	Controller type		
	PID	AT1FPID	AIT2FPID
%OS	15.70	9.78	8.92
$T_s$	164	101	85
IAE	29.05	27.25	26.79
ITAE	640.13	498.32	436.38

Variations in the inflow temperature leading to reservoir temperature changes are represented by Eq. (25) [13].

$$H(s) = \frac{1}{25s+1} e^{-35s} \tag{25}$$

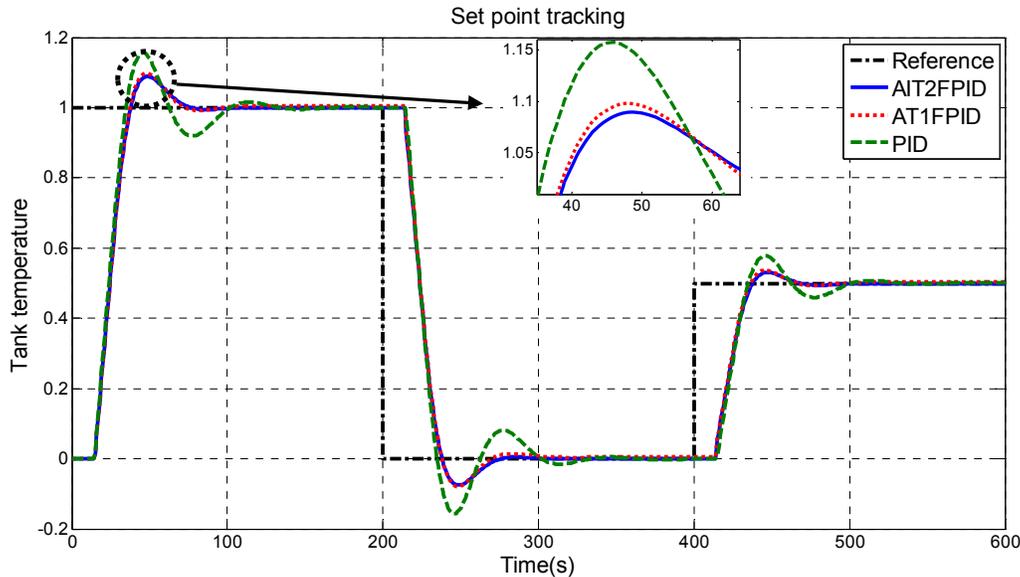


Fig. 10. System response (normalized tank temperature) for set point tracking

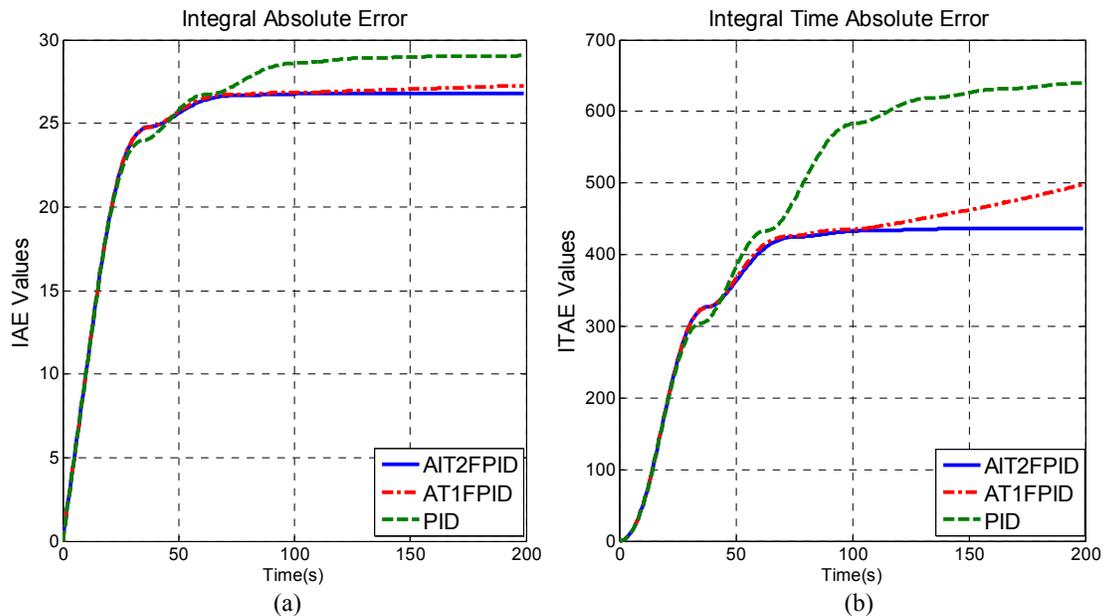


Fig. 11. Tracking performance (a) IAEand (b) ITAE for conventional PID, T1FLS and IT2FLS tuning

Likewise, the simulation results for disturbance rejection by using a step function are illustrated in Fig. 12. Capabilities of AIT2FPID for disturbance rejection in comparison with other methods can be easily inferred. Moreover, the endurance of unforeseen internal disturbances and data uncertainties is effectively better in AIT2FPID.

IAE and ITAE performance evaluation curves and the corresponding statistical investigations are shown in Fig. 13 and Table 6 to make an exact comparison between the controllers in terms of performance and error while performing disturbance rejection.

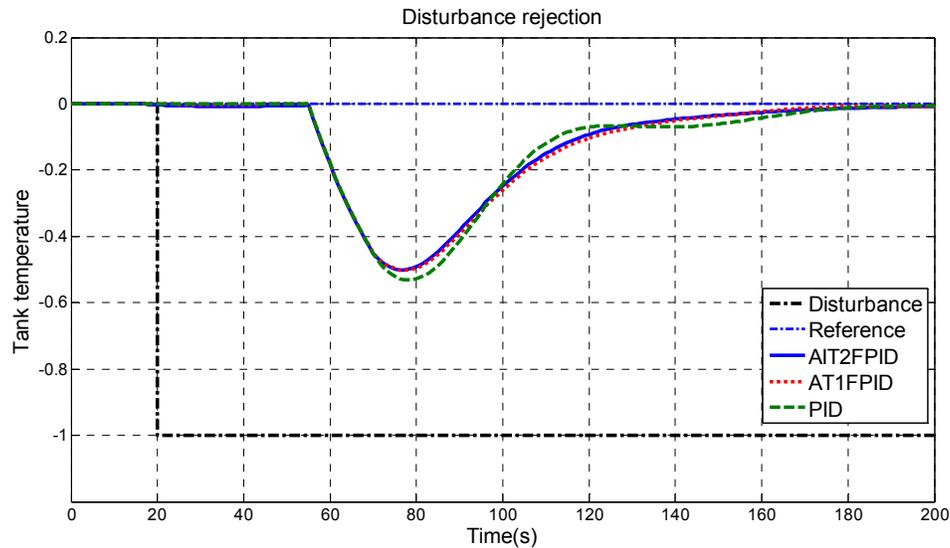


Fig. 12. The Step input and load disturbance response of the plant

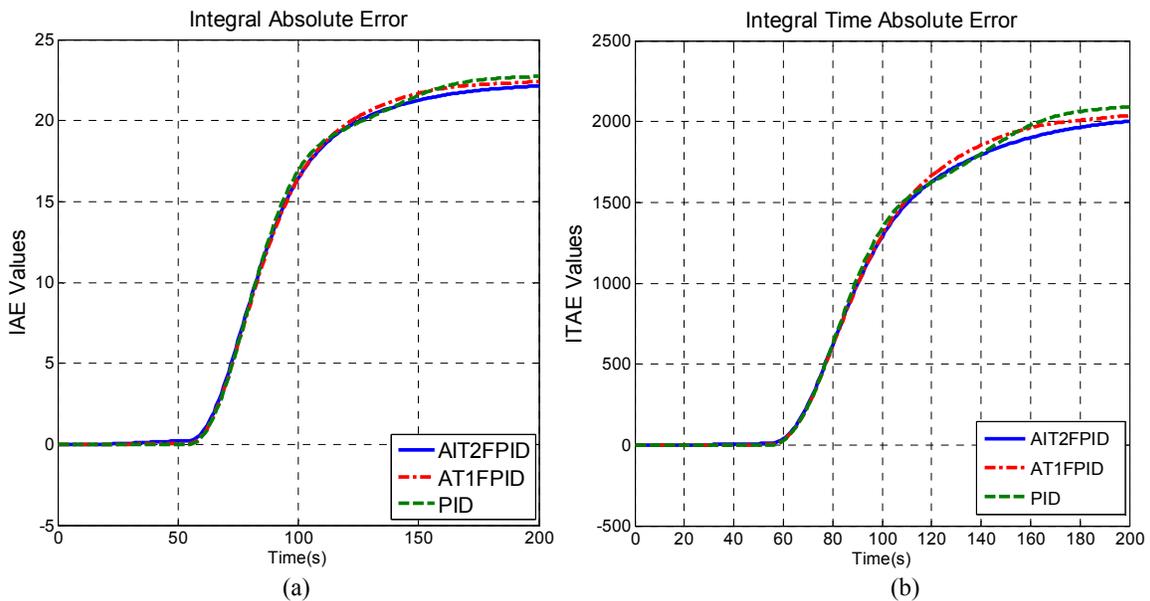


Fig. 13. Step input and load disturbance performance (a) IAE and (b) ITAE for conventional PID, T1FLS and IT2FLS tuning

Table 6. The performance of PID tuning techniques (Disturbance rejection 200 sec)

Performance Criteria	Controller type		
	PID	AT1PID	AIT2PID
IAE	22.89	22.41	22.13
ITAE	2128.45	2037.71	2002.04

It is clear from the error and error derivative that type-2 fuzzy has much better adaptation for the adjustment of PID parameters. Therefore, employing type-2 fuzzy leads to better response of the closed-

loop system and thus the tank temperature has been adjusted quickly while possessing better response characteristics.

In order to assess the proposed control strategy in this study, the results are compared with other conventional control techniques performed on heat exchanger systems already and previously mentioned in the introduction like MPC and FLC. Figure 14 shows the simulation results for set point tracking and Fig. 15 also shows the disturbance rejection results. Based on the error reported for control strategies in Tables 7 and 8, AIT2FPID is obviously more promising than other methods.

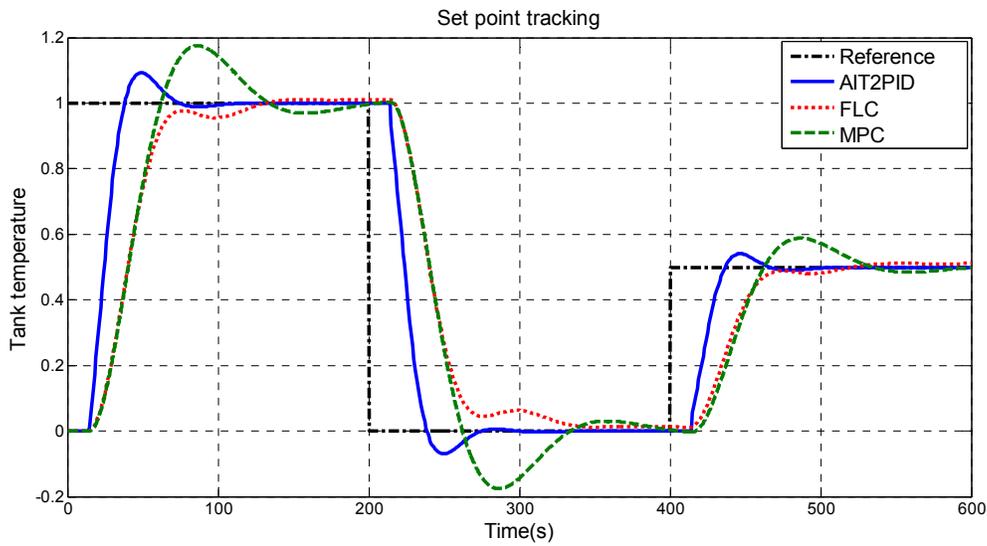


Fig. 14. System response (normalized tank temperature) for set point tracking

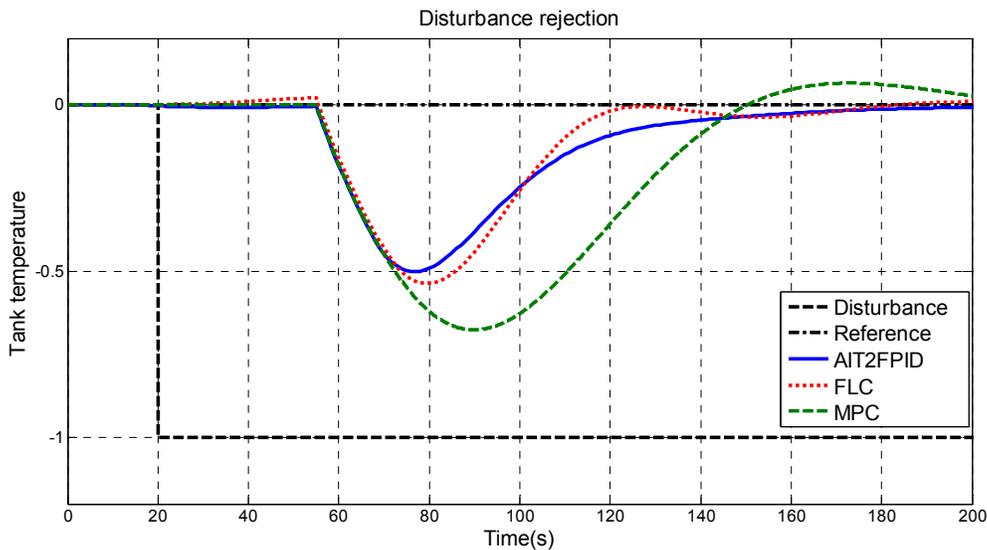


Fig. 15. The Step input and load disturbance response of the plant

Table 7. Performance comparison of AIT2PID, FLC and MPC (Tracking 600 sec)

Performance Criteria	Controller type		
	AIT2PID	FLC	MPC
IAE	46.165	76.621	332.01
ITAE	11027.49	19764.76	111620.72

Table 8. Performance comparison of AIT2PID, FLC and MPC (Disturbance rejection 200 sec)

Performance Criteria	Controller type		
	AIT2PID	FLC	MPC
IAE	22.13	22.530	39.50
ITAE	2002.04	2265.18	3962.91

## 7. CONCLUSION

The application of type-2 fuzzy sets for PID parameter adaptation is investigated in order to suggest a novel method for controlling the temperature of a heat exchanger system. Based on the results, the proposed method is efficient in terms of controller performance and disturbance rejection. Besides, the proposed method is able to accomplish parameter adaptation considering model complexities, uncertainties of the input data and also utilising type-2 fuzzy sets. The simulation results illustrate that this method has better performance for tracking the input signal resulting in response accuracy and a system reaction with smaller errors in comparison with AT1FPID, MPC, conventional FLC and conventional PID controllers. Furthermore, the selected controller is more proficient for harnessing the disturbances arising from inflow to the tank. In addition, the proposed method can be applied to other industrial processes with inherent uncertainties in the input data encountering complexities in order to perform multiple modifications leading to the enhancement of total efficiency.

## REFERENCES

1. Åström, K. J. & Hägglund, T. (2001). The future of PID control. *Control Engineering Practice*, Vol. 9, pp. 1163-1175.
2. Ang, K. H., Chong, G. & Li, Y. (2005). PID control system analysis, design, and technology. *Control Systems Technology, IEEE Transactions on*, Vol. 13, pp. 559-576.
3. Marsh, P. (1998). Turn on, tune in, where can the PID controllers go next. *New Electron*, Vol. 31, pp. 31-32.
4. Rao, B. P. & Voleti, D. (2011). A novel approach of designing fuzzy logic based controller for water temperature of heat exchanger process control. (*IJAEST*) *International Journal of Advanced Engineering Sciences and Technologies*, Vol. 11, pp. 172-176.
5. Vasičkaninová, A. & Bakošová, M. (2011). Intelligent control of heat exchangers. *14th International Conference on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction*.
6. Franklin, G. F., Powell, J. D. & Emami-Naeini, A. (2009). *Feedback control of dynamic systems*, 6 ed.: Prentice Hall.
7. Zhang, X., Lu, K., Li, X. & Xiong, X. (2012). Research on the Modeling and Simulation of Shell and Tube Heat Exchanger System Based on MPCE. *Intelligent Human-Machine Systems and Cybernetics (IHMSC), 4th International Conference on*, pp. 328-331.
8. Tajjudin, M., Rahiman, M. H. F., Ishak, N., Ismail, H., Arshad, N. M. & Adnan, R. (2012). Modified Relay Tuning PI with Error Switching: A Case Study on Steam Temperature Regulation. *International Journal of Engineering Research and Applications (IJERA)*, Vol. 2, pp. 2091-2096.
9. Khan, S. (2014). Modelling and temperature control of heat exchanger process. *International Journal for Research in Applied Science and Engineering Technology*, Vol. 2, pp. 66-74, 2014.
10. Kokate, R. & Waghmare, L. M. (2009). IMC-PID and predictive controller design for a shell and tube heat exchanger. *Emerging Trends in Engineering and Technology (ICETET), 2nd International Conference on*, pp. 1037-1041.
11. Lim, K. & Ling, K. (1989). Generalized predictive control of a heat exchanger. *Control Systems Magazine, IEEE*, Vol. 9, pp. 9-12.

12. Kokate, R. & Waghmare, L. (2010). Cascade generalized predictive control for heat exchanger process. *International Journal of Signal System Control and Engineering Applications*, Vol. 3, pp. 13-27.
13. Duka, A. V. & Oltean, S. E. (2012). Fuzzy control of a heat exchanger. *Automation Quality and Testing Robotics (AQTR), 2012 IEEE International Conference on*, pp. 135-139.
14. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, Vol. 8, pp. 338-553.
15. Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *Systems, Man and Cybernetics, IEEE Transactions on*, pp. 28-44.
16. Castro, J. (1995). Fuzzy logic controllers are universal approximators. *Systems, Man and Cybernetics, IEEE Transactions on*, Vol. 25, pp. 629-635.
17. Yager, R. R. & Kreinovich, V. (2003). Universal approximation theorem for uniform-based fuzzy systems modeling," *Fuzzy Sets Syst*, Vol. 140, pp. 331-339.
18. Al-Khazraji, A., Essounbouli, N., Hamzaoui, A., Nollet, F. & Zaytoon, J. (2011). Type-2 fuzzy sliding mode control without reaching phase for nonlinear system. *Engineering applications of artificial intelligence*, Vol. 24, pp. 23-38.
19. Lee, C. S., Wang, M. H. & Hagra, H. (2010). A type-2 fuzzy ontology and its application to personal diabetic-diet recommendation. *Fuzzy Systems, IEEE Transactions on*, Vol. 18, pp. 374-395.
20. Wang, L. X. (1992). Fuzzy systems are universal approximators. *Fuzzy Systems, 1992., IEEE International Conference on*, pp. 1163-1170.
21. Hagra, H. A. (2004). A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots. *Fuzzy Systems, IEEE Transactions on*, Vol. 12, pp. 524-539.
22. Mencattini, A., Salmeri, M. & Lojacono, R. (2006). Type-2 fuzzy sets for modeling uncertainty in measurement. *Advanced Methods for Uncertainty Estimation in Measurement, 2006. AMUEM 2006. Proceedings of the 2006 IEEE International Workshop on*, pp. 8-13.
23. Wu, D. & Mendel, J. M. (2010). Computing with words for hierarchical decision making applied to evaluating a weapon system. *Fuzzy Systems, IEEE Transactions on*, Vol. 18, pp. 441-460.
24. Castillo, O. & Melin, P. (2008). *Type-2 fuzzy logic: theory and applications*, Vol. 223: Springer.
25. Mamdani, E. H. (1974). Application of fuzzy algorithms for control of simple dynamic plant. *Electrical Engineers, Proceedings of the Institution of*, Vol. 121, pp. 1585-1588.
26. Mamdani, E. H. & Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller. *International journal of man-machine studies*, Vol. 7, pp. 1-13.
27. Wu, D. & Tan, W. W. (2004). A type-2 fuzzy logic controller for the liquid-level process. *Fuzzy Systems, Proceedings. 2004 IEEE International Conference on*, pp. 953-958.
28. Wu Woei Wan Tan, D. (2006). A simplified type-2 fuzzy logic controller for real-time control. *ISA transactions*, Vol. 45, pp. 503-516.
29. Abbadi, A., Nezli, L. & Boukhetala, D. (2013). A nonlinear voltage controller based on interval type 2 fuzzy logic control system for multimachine power systems. *International Journal of Electrical Power & Energy Systems*, Vol. 45, pp. 456-467.
30. Martínez, R., Castillo, O. & Aguilar, L. T. (2009). Optimization of interval type-2 fuzzy logic controllers for a perturbed autonomous wheeled mobile robot using genetic algorithms. *Information Sciences*, Vol. 179, pp. 2158-2174.
31. Melin, P., Astudillo, L., Castillo, O., Valdez, F. & Garcia, M. (2013). Optimal design of type-2 and type-1 fuzzy tracking controllers for autonomous mobile robots under perturbed torques using a new chemical optimization paradigm. *Expert Systems with Applications*, Vol. 40, pp. 3185-3195.
32. Castillo, O., Huesca, G. & Valdez, F. (2005). Evolutionary computing for optimizing type-2 fuzzy systems in intelligent control of non-linear dynamic plants. *Fuzzy Information Processing Society, NAFIPS 2005. Annual Meeting of the North American*, pp. 247-251.

33. Liao, Q., Li, N. & Li, S. (2009). Type-II TS fuzzy model-based predictive control. *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pp. 4193-4198.
34. Lin, T. C. (2010). Observer-based robust adaptive interval type-2 fuzzy tracking control of multivariable nonlinear systems. *Engineering Applications of Artificial Intelligence*, Vol. 23, pp. 386-399.
35. Chafaa, K., Saidi, L., Ghanai, M. & Benmahammed, K. (2007). Indirect adaptive interval type-2 fuzzy control for nonlinear systems. *International Journal of Modelling, Identification and Control*, Vol. 2, pp. 106-119.
36. Lin, T. C., Liu, H. L. & Kuo, M. J. (2009). Direct adaptive interval type-2 fuzzy control of multivariable nonlinear systems. *Engineering Applications of Artificial Intelligence*, Vol. 22, pp. 420-430.
37. Karnik, N. N. & Mendel, J. M. (1998). *An introduction to type-2 fuzzy logic systems*. Technical Report.
38. Mousavi, S., Hashemipour, M., Sadeghi, M., Petrofsky, J. & Prowse, M. (2010). A fuzzy logic control system for the rotary dental instruments. *Iranian Journal of Science and Technology Transaction B: Engineering*, Vol. 34, pp. 539-551.
39. Li, H. X., Zhang, L., Cai, K. Y. & Chen, G. (2005). An improved robust fuzzy-PID controller with optimal fuzzy reasoning. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, Vol. 35, pp. 1283-1294.
40. Mohan, B. & Sinha, A. (2008). Analytical structure and stability analysis of a fuzzy PID controller. *Applied Soft Computing*, Vol. 8, pp. 749-758.
41. Hu, B. G., Mann, G. K. I. & Gosine, R. G. (2001). A systematic study of fuzzy PID controllers-function-based evaluation approach. *Fuzzy Systems, IEEE Transactions on*, Vol. 9, pp. 699-712.
42. Wang, H. X., Ying, M. F. & Zai, L. X. (2013). Design of fuzzy PID controller based on adaptive particle swarm optimization with cloud theory. *Advanced Materials Research*, Vol. 706, pp. 720-723.
43. Kokate, R. D. & Waghmare, L. M. (2009). Research and application of new dynamic matrix control based on state feedback theory for heat exchanger control. *Emerging Trends in Engineering and Technology (ICETET), 2009 2nd International Conference on*, pp. 1031-1036.
44. Smith, C. A. & Corripio, A. B. (1985). *Principles and practice of automatic process control*. Vol. 2, Wiley New York.
45. Mendel, J. M. & John, R. I. B. (2002). Type-2 fuzzy sets made simple. *Fuzzy Systems, IEEE Transactions on*, Vol. 10, pp. 117-127.
46. Leal-Ramírez, C., Castillo, O., Melin, P. & Rodríguez-Díaz, A. (2011). Simulation of the bird age-structured population growth based on an interval type-2 fuzzy cellular structure. *Information Sciences*, Vol. 181, pp. 519-535.
47. Karnik, N. N., Mendel, J. M. & Liang, Q. (1999). Type-2 fuzzy logic systems. *Fuzzy Systems, IEEE Transactions on*, Vol. 7, pp. 643-658.
48. Liang, Q. & Mendel, J. M. (2000). Interval type-2 fuzzy logic systems: theory and design. *Fuzzy Systems, IEEE Transactions on*, Vol. 8, pp. 535-550.
49. Mendel, J. M. (2007). Type-2 fuzzy sets and systems: an overview. *Computational Intelligence Magazine, IEEE*, Vol. 2, pp. 20-29.
50. Jammeh, E. A., Fleury, M., Wagner, C., Hagrass, H. & Ghanbari, M. (2009). Interval type-2 fuzzy logic congestion control for video streaming across IP networks. *Fuzzy Systems, IEEE Transactions on*, Vol. 17, pp. 1123-1142.
51. Castillo, O. (2012). *Type-2 fuzzy logic in intelligent control applications*: Springer.
52. Karnik, N. N. & Mendel, J. M. (2001). Centroid of a type-2 fuzzy set. *Information Sciences*, Vol. 132, pp. 195-220.
53. Tang, K., Man, K. F., Chen, G. & Kwong, S. (2001). An optimal fuzzy PID controller. *Industrial Electronics, IEEE Transactions on*, Vol. 48, pp. 757-765.