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ANALYSIS OF COMBINED RADIATIVE AND CONDUCTIVE HEAT TRANSFER IN THREE-DIMENSIONAL COMPLEX GEOMETRIES USING BLOCKED-OFF METHOD*

K. LARI AND S. A. GANDJALIKHAN NASSAB**
Dept. of Mechanical Engineering, School of Engineering, Shahid Bahonar University, Kerman, I. R. of Iran
Email: ganj110@mail.uk.ac.ir

Abstract– This paper presents a simple Cartesian practical technique named blocked-off-region procedure to study the combined conductive and radiative heat transfer in three-dimensional irregular geometries. By a concept of blocked-off method previously applied in the problems of computational fluid dynamics, both straight and curvilinear boundaries can be treated. The set of equations consisting of gas energy equation and also the radiative transfer equation are solved simultaneously to obtain the temperature and radiative heat flux distributions inside the participating medium. The finite volume method has been adopted to solve the energy equation and the discrete ordinates method (DOM) is used to model the radiative transfer in an absorbing-emitting and linear anisotropic scattering medium. The radiative conductive model is validated by comparison with test cases solutions from the literature and is applied to analyze the effect of thermoradiative parameters such as conduction-radiation parameter, optical thickness and scattering albedo on the temperature and radiative flux distributions for three-dimensional enclosures. Results confirm that the proposed method is a good general way for studying combined conductive and radiative heat transfer in three-dimensional complex enclosures.

Keywords– Three-dimensional, complex geometries, radiation, conduction, blocked-off method, discrete ordinates method

1. INTRODUCTION

Heat transfer by coupled conduction and radiation in an absorbing, emitting and scattering medium is important in many areas such as heat transfer in glass fabrication and in thermal insulation materials. These insulations can be used in cryogenic and space applications, fluidized beds, optical engineering, solar engineering, nuclear engineering, and so on. Due to the difficulty in finding the exact analytical solution to integro-differential radiative transfer equation (RTE) in participating media, a variety of numerical methods have been worked out over the last few decades. They include the zone method, Monte Carlo method, flux method [1], discrete transfer method, P_N method, discrete ordinates method (DOM), finite volume method (FVM), etc. Each of these methods has its own relative advantages and disadvantages, and none of them is superior to others in all aspects.

To be embedded in a comprehensive numerical method, a suitable discretization method for radiation should: (1) permit compatibility with available methods that solve fluid flow and convective heat transfer; (2) allow geometrical flexibility; (3) account for absorption, emission, and anisotropic scattering; and (4) be affordable. In this respect, there exist two competing methods that share many researchers interest: the Discrete Transfer Method (DTM) and the Discrete Ordinates Method (DOM), and their variations and

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**Corresponding author

improvements [2]. The method of discrete ordinates DOM is an attractive simplified method to solve radiative transfer problems. It offers a good compromise between accuracy and computational requirements. In particular, the DOM was originally formulated by Chandrasekar in 1950 [3], and has been deeply studied by Carlson and Lathrop in the 1960-70's [4] and by Truelove in the 80's [5].

Among the few earlier works on combined conduction-radiation problems, Razzaque et al. [6] analyzed the two-dimensional coupled conduction-radiation problems with finite element method limited to only non-scattering medium with isothermal black walls. The product integration method found its application in the work of Tan [7] for solving combined conduction-radiation problem in square enclosure with isothermal walls. Kim et al. [8] employed the discrete ordinates method (DOM) for coupled radiative and conductive heat transfer in rectangular enclosures. Rousse [2] and Rousse et al. [9] used the Control-Volume Finite Element Method (CVFEM) for the solution of combined mode of heat transfer in two-dimensional cavities. Collapse dimension method has been used to analyse combined conduction-radiation problem by Talukdar et al. [10]. Mahapatra et al. [11] investigated a new hybrid method where the concepts of modified differential approximation were employed by blending discrete ordinate method and spherical harmonics method on combined heat transfer in two-dimensional planar geometry. Recently, Amiri et al. [12] analyzed the problem of combined conduction and radiation heat transfer in 2D irregular geometries by using DOM and blocked-off method with both temperature and heat flux boundary conditions.

Irregular geometries can be modeled using body-fitted grid system with nonorthogonal grids. However, additional complexity arises from the nonorthogonality of the computational grids. It is therefore desirable to formulate a procedure to model irregular geometries using Cartesian coordinates formulation. The concept of blocked-off region was previously applied in the computational fluid dynamics (CFD) [13] problems and then extended to two-dimensional radiative transfer problem by Chai et al. [14, 15]. They found very promising results for different 2D problems. Coelho et al. [16] used this method with the FVM and the DOM to predict radiative heat transfer in enclosures containing obstacles of very small thickness (baffles). Comparison between three types of boundary treatments, the spatial multiblock, blocked-off, and embedded boundary has been investigated by Byun et al. [17] for different cases of two-dimensional complex enclosures. Talukdar [18] analysed the two-dimensional irregular geometries with the concept of blocked-off region using the Discrete Transfer Method (DTM). He found that the method of blocked-off can be recommended as a good alternative to solve problems with irregular geometries.

Although there is a fast growth of research activity in this heat transfer area, to the best of the authors' knowledge, the blocked-off method has not been employed for analyzing the combined conduction-radiation heat transfer in three-dimensional irregular enclosures using DOM. To reach this goal, the model that we will present is a three-dimensional formulation of blocked-off technique. By using blocked-off method, both straight and curvilinear boundaries can be treated. Also, with this method we can use the same algorithms which are employed in regular geometries for enclosures with curved and complex boundaries.

In the present study, a three-dimensional blocked-off-region procedure was offered to model combined conductive and radiative heat transfer in complex geometries. The radiative heat transport equation is solved using the conventional procedure of solution for DOM in an absorbing-emitting and isotropic or linear anisotropic scattering medium. The finite volume method has been adopted to solve the energy equation and the radiative source term in the energy equation is computed from intensities field. The radiative conductive model is validated by comparison with the well-documented results in literature along three test cases. Besides, this model is applied to analyze the effect of the main thermoradiative parameters on the temperature and radiative flux distributions in three-dimensional enclosures.

2. FORMULATION

The energy equation for coupled radiation–conduction heat transfer of an absorbent, emitter and scatter media under steady state condition with constant thermal conductivity is as in Siegel and Howell [19]:

$$k\nabla^2 T - \nabla . q_r = 0 \tag{1}$$

To obtain the temperature distribution in the medium by solving Eq. (1), it is necessary to relate $\nabla . q_r$ to the temperature distribution within the medium. One approach is to obtain $\nabla . q_r$ directly by considering the local radiative interaction with a differential volume in the medium. The local divergence of the radiative flux is related to the local intensities by

$$\nabla . q_r = \kappa\left[4\pi I_b(r) - G(r)\right] \tag{2}$$

where $G(r)$ is incident radiation

$$G(r) = \int_{4\pi} I(r, \Omega) d\Omega \tag{3}$$

To obtain the radiation intensity field and $\nabla . q_r$, it is necessary to solve the radiative transport equation (RTE). This equation for an absorbing, emitting and scattering gray medium can be written as in Modest [20],

$$(\Omega . \nabla) I(r, \Omega) = -\beta I(r, \Omega) + \kappa I_b(r) + \frac{\sigma_s}{4\pi} \int_{4\pi} I(r, \Omega') \Phi(\Omega, \Omega') d\Omega' \tag{4}$$

where $I(r, \Omega)$ is the radiation intensity in position r, and in the direction Ω, $I_b(r)$ is the radiation intensity of the blackbody in the position r and at the temperature of the medium, κ and σ_s are the gray medium absorption and scattering coefficients, respectively, $\beta = (\kappa + \sigma_s)$ is the extinction coefficient, and $\Phi(\Omega, \Omega')$ is the scattering phase function for radiation from incident direction Ω' to scattered direction Ω, and the integration is in the incident direction.

For diffusely reflecting surfaces, the radiative boundary condition for Eq. (4) is

$$I(r, \Omega) = \varepsilon I_b(r) + \frac{\rho}{\pi} \int_{n.\Omega' < 0} I(r, \Omega') |n.\Omega'| d\Omega' \tag{5}$$

where r belongs to the boundary surface and Eq. (5) applies for $n.\Omega > 0$, $I(r, \Omega)$ is the radiation intensity leaving the surface at the boundary condition, ε is the surface emissivity, ρ is the surface reflectivity, and n is the unit vector normal to the boundary surface.

In the method of discrete ordinates, the equation of radiation transport is substituted by a set of *M* discrete equations for a finite number of directions Ω_m, and each integral is substituted by a quadrature series of the form,

$$(\Omega_m . \nabla) I(r, \Omega_m) = -\beta I(r, \Omega_m) + \kappa I_b(r) + \frac{\sigma_s}{4\pi} \sum_{k=1}^{M} w_k I(r, \Omega_k) \Phi(\Omega_m, \Omega_k) \tag{6}$$

subject to the boundary conditions

$$I(r, \Omega_m) = \varepsilon I_b(r) + \frac{\rho}{\pi} \sum_{n.\Omega_k < 0} w_k I(r, \Omega_k) |n.\Omega_k| \tag{7}$$

where w_k are the quadrature weights. This angular approximation transforms the original equation into a set of coupled differential equations.

The incident radiation (Eq. (3)) in discrete ordinates will be,

$$G(r) = \sum_{k=1}^{M} w_k I(r, \Omega_k) \tag{8}$$

3. METHOD OF SOLUTION OF RADIATIVE CONDUCTIVE MODEL

To solve the coupled radiation–conduction problem, the Eqs. (1) and (2) are converted to non-dimensional form,

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} = S_r \tag{9}$$

$$S_r = \frac{\tau^2 (1-\omega)}{N_{cr}} \left(\theta^4(r) - \frac{1}{4} \sum_{k=1}^{M} w_k \bar{I}(r, \Omega_k) \right) \tag{10}$$

by employing a reference length, L_*, a reference absolute temperature, T_*, and the following dimensionless variables,

$$\theta = \frac{T}{T_*},\; X = \frac{x}{L_*},\; Y = \frac{y}{L_*},\; Z = \frac{z}{L_*},\; \tau = \beta L_*,\; \bar{I} = \frac{I}{\sigma T_*^4},\; N_{cr} = \frac{k\beta}{4\sigma T_*^3},\; Q = \frac{q}{\sigma T_*^4} \tag{11}$$

where N_{cr} is the Conduction-Radiation parameter (the Stark number, Rousse [2]).

After solving the intensities and temperature fields, the heat fluxes can be calculated. The dimensionless directional heat fluxes, Q_x, Q_y and Q_z, consisting of both conduction and radiation are defined as in Kim et al. [8]:

$$\begin{aligned} Q_x &= -\frac{4N_{cr}}{\tau} \frac{\partial \theta}{\partial X} + \sum_{k=1}^{M} w_k \xi_k \bar{I}_k \\ Q_y &= -\frac{4N_{cr}}{\tau} \frac{\partial \theta}{\partial Y} + \sum_{k=1}^{M} w_k \eta_k \bar{I}_k \\ Q_z &= -\frac{4N_{cr}}{\tau} \frac{\partial \theta}{\partial Z} + \sum_{k=1}^{M} w_k \mu_k \bar{I}_k \end{aligned} \tag{12}$$

where the first and second terms on the right-hand side are the dimensionless conductive and radiative heat fluxes.

The procedure of the numerical calculations starts by assuming that the radiative source term is zero and the energy equation is solved to find the temperature field. The line by line TDMA Algorithm is used to quickly bring the information from all boundaries to the interior. Then the radiative transport equation is solved using DOM until convergence of the intensities field is obtained. Consequently, the radiative source term is calculated and the energy equation is solved including the source term only once for the global iterative process to permit faster convergency.

The iterative process continues until achieving convergence of the intensities and the temperature field. The convergence of the solution was evaluated using a convergence criterion taken as the error in the intensity field in RTE solution and the error in the temperature field for each node p, respectively, by the following criteria:

$$
\begin{aligned}
error \quad I &= Max\left|\frac{\bar{I}_p^n - \bar{I}_p^{n-1}}{\bar{I}_p^n}\right| \le 10^{-6} \\
error \quad T &= Max\left|\frac{\theta_p^n - \theta_p^{n-1}}{\theta_p^n}\right| \le 10^{-6}
\end{aligned}
\tag{13}
$$

4. THE BLOCKED-OFF METHOD

In order to avoid the complexity of treating non-orthogonal grids, it is suitable to formulate a procedure to model irregular geometries using Cartesian coordinates formulation. The blocked-off method consists of drawing nominal domains around given physical domains, so the region is divided into two parts: active and inactive or blocked-off regions. With this method, we can use the same 3D cubic algorithm to handle curved or inclined boundaries.

In Fig. 1a and b, two sample 2D geometries (for better observation) are presented to show how they are treated to simulate from a rectangular geometry. In these figures, the shaded portion is called inactive or blocked-off region, and is added to real domain to convert irregular geometry to rectangular shaped geometry. The remaining portion is the active region which is the real domain of interest. The rectangular domain can be termed the simulated domain. Curved boundaries appear to be a stair step as shown in Fig. 1b.

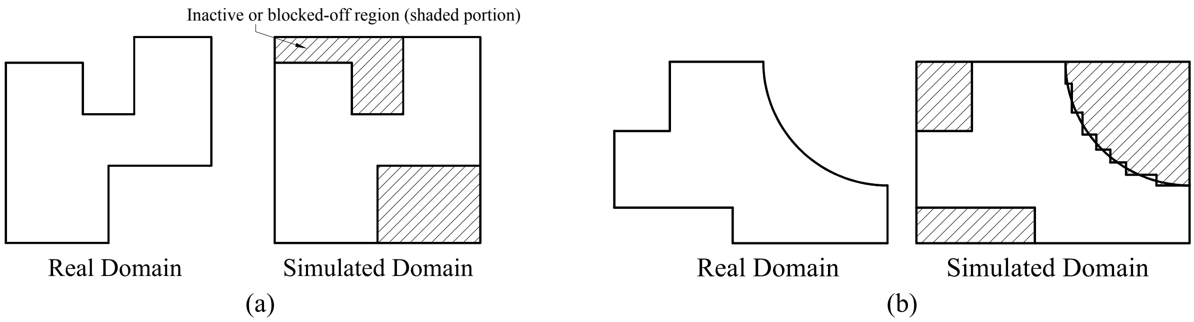


Fig. 1. Sample irregular geometries

The calculation is done over the whole domain, but in inactive regions, solutions are not sought. Thus during the calculations, the magnitude of the quantities such as temperature and intensity at the cells of these regions becomes zero. In order to consider this procedure in solution strategy, for every type of geometry, a domain file has to be created according to Fig. 2. This case is in reference to Fig. 1a. The whole simulated domain is discretised into several control volumes and the control volumes which are inside the active region are intended as one (1), otherwise they are zero (0). Then, during the solution, the magnitude of quantities at the control volumes with zero value in the domain file is considered as zero.

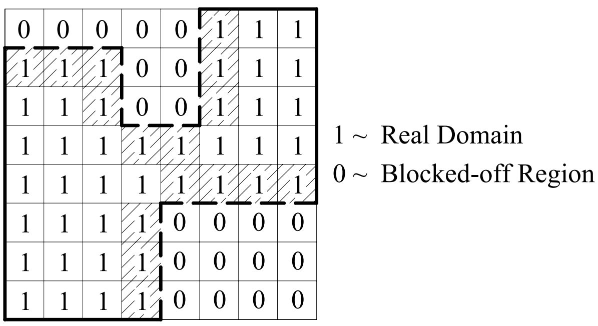


Fig. 2. A domain with blocked-off/inactive region

The other important additional task for this approach is to define the additional boundary condition referred to as the second boundary condition for those real boundaries that, due to the expansion of irregular geometry into the rectangular shape are located inside the computational domain (dashed lines in Fig. 2). For these boundaries, depending on the shape of the geometry, a boundary condition file is specified that includes the control volumes that are adjacent to these internal boundaries and are in the active region (hatched cells in Fig. 2). Then, in the calculations, these boundaries are simply treated like other boundaries (external boundaries which are sides of the rectangular domain).

5. VALIDATION

To show the validity and the accuracy of the current method, different test problems are considered and compared with the available results of the literature. First, a test case is presented to show the validity of the blocked-off method in three dimensional radiation problems and then another test case is given for coupled radiation-conduction problem.

a) Test problem 1: The blocked-off method

As an irregular geometry, we considered an L-shaped enclosure containing an emitting-absorbing medium at a temperature of $1000\,\mathrm{K}$ (see Fig. 3), where the walls are black at $500\,\mathrm{K}$. The grid used is comprised $12 \times 12 \times 5$ control volumes, with $8 \times 8 \times 5$ control volumes in the blocked-off region. In this enclosure, only radiation heat transfer takes place and it is assumed that free convection does not occur in the medium.

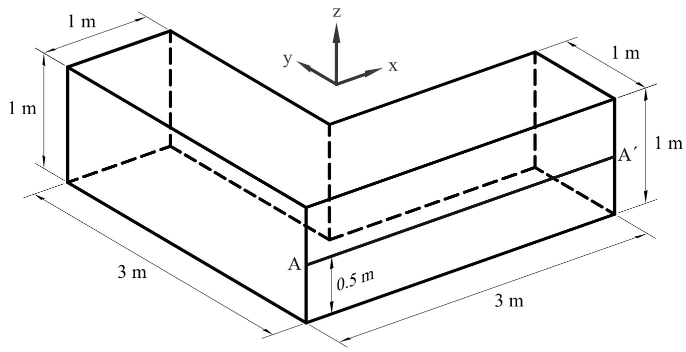


Fig. 3. The geometry of test problem 1

Figure 4 shows the effect of the absorption coefficient of the medium on the predicted net heat flux along the AA' line (marked on Fig. 3). The results obtained with the step scheme have been compared with those presented by Sakami et al. [21] (a new three-dimensional algorithm based on the discrete-ordinates method and using an S_4 quadrature with the exponential spatial discretization scheme applied to a grid include 2000 tetrahedra) and by Joseph et al. [22] (standard discrete ordinates method applied to a non-orthogonal structured grid with the S_4 angular quadrature and the step differencing scheme). Figure 4 shows a good consistency between the present results with those obtained in [21] and [22]. It can be found from this figure that the effect of the left-hand branch of the L-shaped becomes less important as the absorption coefficient increases.

b) Test problem 2: Conduction-radiation

A test case of radiation conduction in a square cavity with participating media is solved in this section. Figure 5 shows a simplified scheme of the cavity with $L_x = L_y = 1\,\mathrm{m}$. The left side surface is hot at temperature $\theta = 1$ and other surfaces are cold at temperature $\theta = 0.5$. The media in cavity is gray and the boundaries are black surfaces. The medium is further assumed to absorb and emit radiation with

$\kappa = 1\,\mathrm{m}^{-1}$, but not scatter radiant energy, $\omega = 0$. In order to use the present three-dimensional algorithm to solve this two-dimensional problem, a cube is considered with the mentioned square cavity as its cross section in *x*-*y* direction and with $L_z = 10\,\mathrm{m}$.

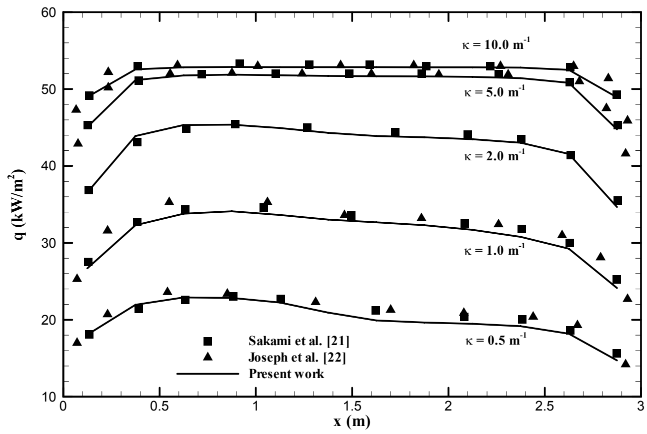


Fig. 4. Net heat flux along line AA' of the L-shaped enclosure (Fig. 3); comparison with the results of Sakami et al. [21] and Joseph et al. [22] $(\omega = 0, \varepsilon = 1.0)$

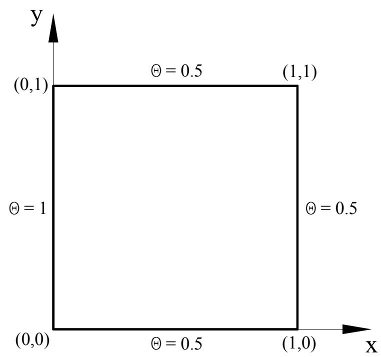


Fig. 5. The geometry of test problem 2

This problem was solved by other investigators using different methods. Tan [7], who treated the radiation problem by the product integration method, analyzed this problem. Also, Kim et al. [8] solved the same problem using the DOM with S_4 angular quadrature, diamond spatial scheme and uniform grid for the radiative part of the problem, while the conductive term is discretized using the central difference scheme. Sakami et al. [23] in their solution of the above problem used a modified discrete ordinates method based on the incorporation of directional ray propagation relations within the cells with triangular grids and in the conduction part of the coupled problem, they used the finite element technique. In a similar way, Rousse et al. [9] used the control volume method in the radiative part coupled with the finite element method in the energy equation and adopted triangular grids.

In Fig. 6, the influence of the conduction-radiation parameter N_{cr} on the dimensionless temperature at the symmetry line $(y = L_y/2,\ z = L_z/2)$ is presented. A finite volume solution is also given for the case of no radiation, $(N_{cr} = \infty)$. When $N_{cr} = 1$, the temperature profile is not greatly affected by radiation when compared to the case of pure conduction. This suggests that N_{cr} should be equal or smaller than unity $(N_{cr} \le 1)$ to require a radiation heat transfer calculation. As N_{cr} decreases, radiation plays a more significant role than conduction. Therefore, as N_{cr} decreases, a steeper temperature gradient is formed at both end walls (right and left surfaces) and the medium temperature inside increases as shown in Fig. 6. The results for the centerline temperature profile are in very good agreement with those presented by Rousse et al. [9].

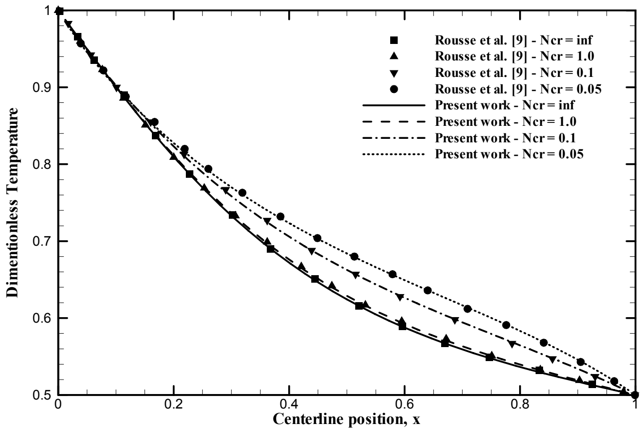
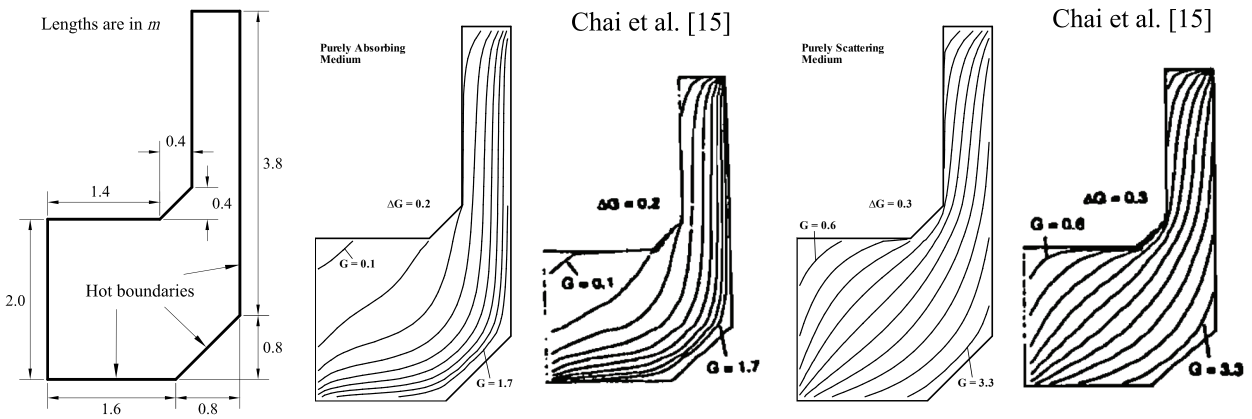


Fig. 6. Centerline temperature distribution in a square black enclosure with absorbing media and different values of the Stark number, N_{cr}; comparison with the results of Rousse et al. [9] $(\omega = 0, \tau = 1)$

6. RESULTS

In this section, the blocked-off region procedure is applied to a three-dimensional J-shaped enclosure. The calculations have been carried out for several cases to investigate the effects of conduction-radiation parameter N_{cr}, scattering albedo ω and the optical thickness τ on thermal behavior of the system.

Figure 7a shows the schematic of a two-dimensional black, J-shaped enclosure studied by Chai et al. [15] and Talukdar et al. [24] that will be used as the cross section of three-dimensional J-shaped enclosure. Firstly, the test case that has been examined in the work of Chai et al. [15] is analyzed. In this case, the south and east walls are hot $\left(E_b = 1\,\mathrm{W/m^2}\right)$, while the medium and the other walls are cold. The medium absorbs and scatters energy isotropically. The computations are performed using 31×41 control volumes. Figures 7b and 7d show contours of incident radiation G for an isotropically scattering medium with $\beta = 1\,\mathrm{m}^{-1}$ and scattering albedos of zero and unity, respectively. In Figs. 7c and 7e the same results of Chai et al. [15] are shown for comparison to the Figs. 7b and 7d, respectively. As it can be seen, our plots are similar to the plots of Chai et al. [15].



(a) (b) (c) (d) (e)

Fig. 7. (a) Schematic of a J-shaped enclosure, (b) Incident radiation for a purely absorbing medium, (c) Same as (b) but from Chai et al. [15], (d) Incident radiation for a purely scattering medium, (e) Same as (d) but from Chai et al. [15] $\left(\beta = 1\,\mathrm{m}^{-1}\right)$

The schematic of the three-dimensional J-shaped enclosure with its simulated domain is shown in Fig. 8a. The dimensions of the three-dimensional J-shaped enclosure are the same as the dimensions shown in

Fig. 7a with $L_z = 2\,\text{m}$. In the following test cases, combined conduction-radiation situation is studied. All walls are black where the wall $x = 0$ is at dimensionless temperature $\theta = 1$ and the others are at 0.5. For all cases, we have considered $31 \times 41 \times 11$ spatial mesh.

In the first test case, the medium is considered to be non-scattering with $\tau = 1$. Figure 8b shows the dimensionless temperature along the *x*-direction at the line AA' of the J-shaped enclosure (Fig. 8a) for various values of the conduction-radiation parameter N_{cr}. It is obvious that when N_{cr} increases the conductive heat transfer mode becomes dominant until it reaches infinity, when, in this point, the energy equation will transform into purely conduction heat transfer. In an opposing manner, when N_{cr} becomes zero, the medium will be at purely radiative heat transfer situation. It is seen from Fig. 8b that as N_{cr} decreases, a steeper temperature gradient is formed at both $x = 0$ and $x = L_x$ walls and the medium temperature far from the hot surface increases. In fact, the radiative energy emitted from the hot wall can penetrate more deeply into the medium and is therein transformed into thermal energy (Kim et al. [8]).

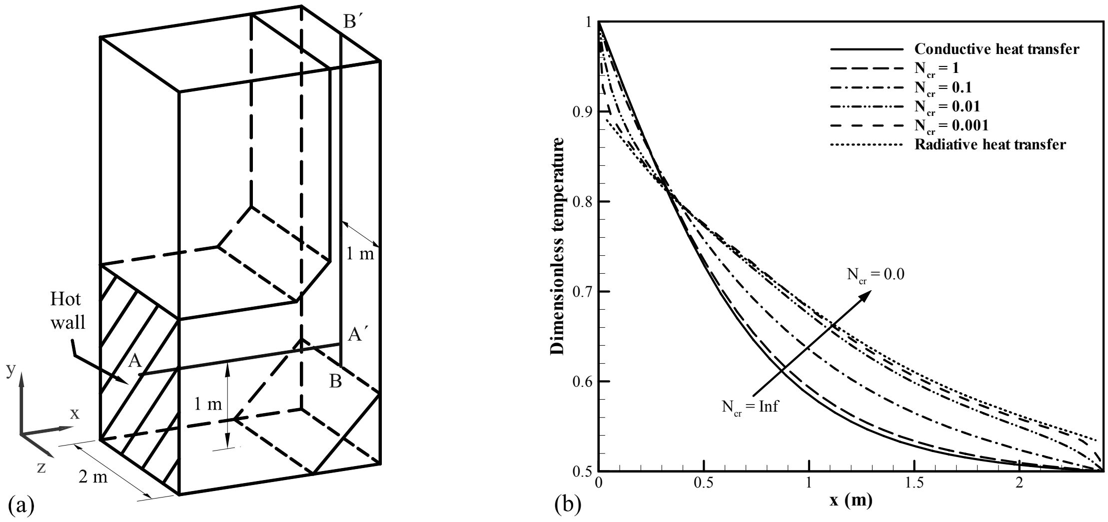


Fig. 8. (a) Schematic of the 3D J-shaped enclosure with simulated domain, (b) Dimensionless temperature profiles at line AA' for various conduction-radiation parameters N_{cr} $(\omega = 0, \tau = 1)$

The dimensionless total heat flux and the wall radiative heat flux to total heat flux ratio (the fractional radiative heat flux) along the line BB' of the J-shaped enclosure (Fig. 8a) for different values of N_{cr} are shown in Fig. 9. Fig. 9a shows that the total flux becomes clearly much higher as N_{cr} increases and more uniform as N_{cr} decreases. According to Fig. 9b, the fractional radiative heat flux is seen to increase when N_{cr} decreases. Also, it is seen that when N_{cr} is large and conductive heat transfer mode is dominant, the radiative transfer mode is greatly effective in the upper half of the enclosure.

In the following, we fixed the conduction-radiation parameter at $N_{cr} = 0.1$ and an absorbing-emitting and non-scattering medium is considered. The dimensionless total flux and the fractional radiative heat flux along the line BB' are shown in Fig. 10 for various values of the optical thicknesses τ. The rise in optical thickness is caused by a rise in extinction coefficient β, and when $\omega = 0$ it is caused by a rise in absorption coefficient κ. Therefore, when τ increases a significant amount of radiative heat flux entered from the hot wall is absorbed by the medium, causing a decrease in the radiative fluxes exited from other walls. Also, an increase in τ and therefore an increase in β leads to a decrease in thermal conductivity k when N_{cr} is constant, causing decrease in the wall conductive fluxes. Thus, when τ increases the total heat flux decreases (Fig. 10a). Fig. 10b shows that the fractional radiative heat flux increases with decreasing τ. Due to the fact that decrease in τ causes an increase in both radiative and conductive heat fluxes, the results of Fig. 10b confirm that the radiative flux has much higher increase than conductive flux.

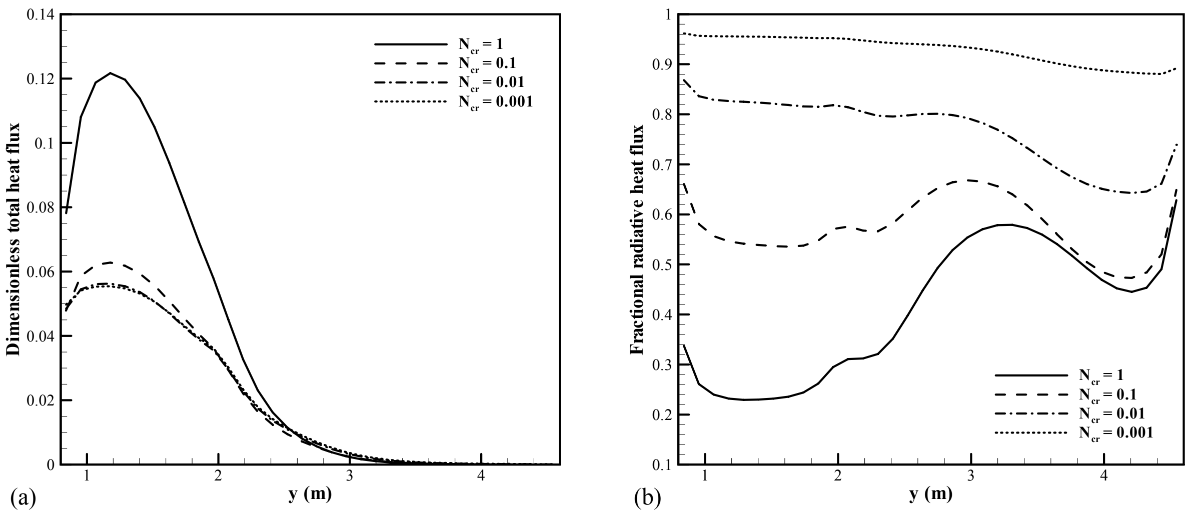


Fig. 9. (a) Dimensionless total heat flux; and (b) fractional radiative heat flux at line BB' for various conduction-radiation parameters N_{cr} $(\omega = 0, \tau = 1)$

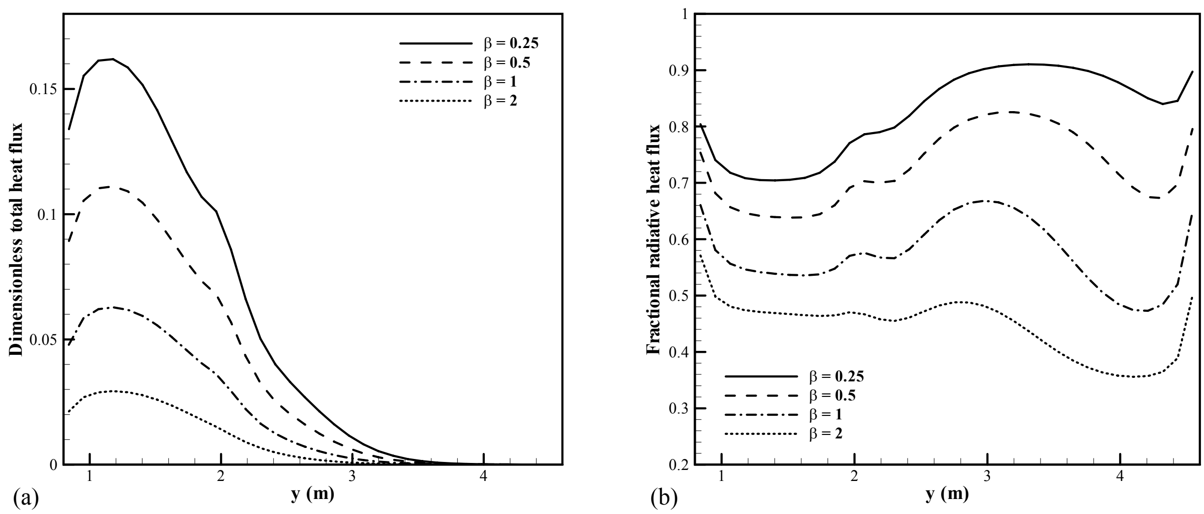


Fig. 10. (a) Dimensionless total heat flux; and (b) fractional radiative heat flux at line BB' for various extinction coefficients β $(\omega = 0, N_{cr} = 0.1)$

Figure 11 shows the dimensionless total flux and the fractional radiative heat flux at the line BB' for various scattering albedo values with $N_{cr} = 0.1$ and $\tau = 1$. The fractional radiative heat flux is seen to increase as ω increases (Fig. 11b), while according to Fig. 11a it seems that the dimensionless total heat flux is insensitive to this parameter. When τ is constant and therefore extinction coefficient β is constant, increasing ω causes increase in scattering coefficient σ_s and decrease in absorption coefficient κ. Thus, when ω increases and other parameters are constant, the radiative heat flux exited from the line BB' increases (Fig. 11b). In the case of purely scattering medium $(\omega = 1)$, the divergence of the radiative heat flux becomes zero $(\nabla . q_r = 0)$, and as stated in Eq. (1), the energy equation is converted into purely conductive heat transfer. Thus, the obtained results in the case of $\omega = 1$ are the same as the results of purely conduction situation.

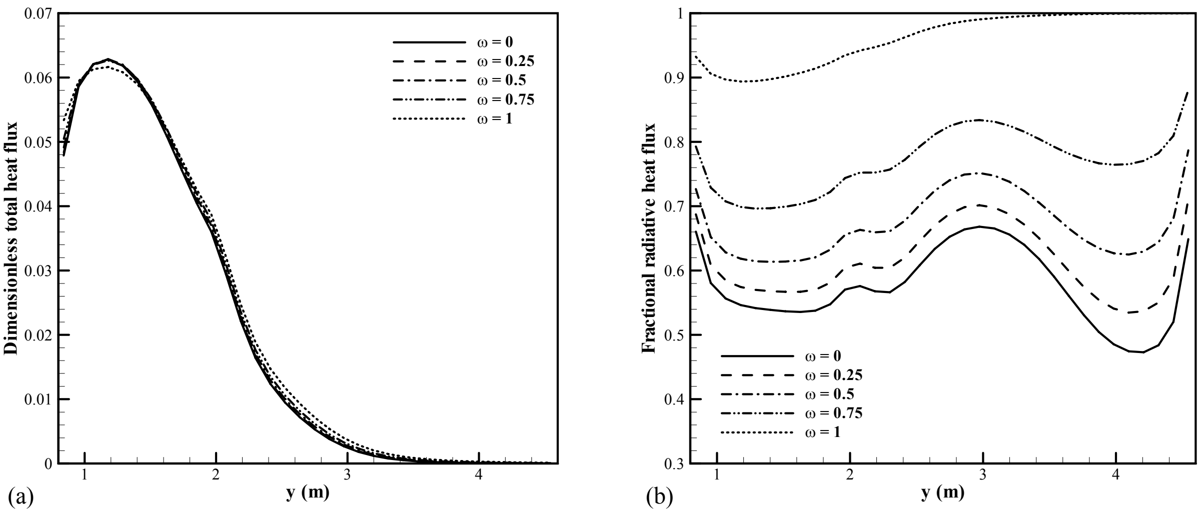


Fig. 11. (a) Dimensionless total heat flux; and (b) fractional radiative heat flux at line BB' for various scattering albedos ω $\left(N_{cr}=0.1, \tau=1\right)$

7. CONCLUSION

An original 3D combination of the discrete ordinates method and the blocked-off-region technique was developed for analysis of combined conduction-radiation heat transfer problems. The blocked-off method that is used in this work is simple and useful for handling combined heat transfer problems in Cartesian space and can be utilized coupled with common control volume algorithms used in fluid dynamics. Also, this method has the major advantage of using the same rectangular algorithm for all types of geometries.

In the first part of this work, the discrete ordinates method formulated with the blocked-off region procedure was developed and tested in three-dimensional complex enclosures with diffuse reflective surfaces and containing gray absorbing-emitting and scattering medium. Compared with benchmarked results, the discrete ordinates method gives satisfactory predictions of both wall heat flux and temperature distributions. It is evident that for curved or inclined boundaries, a fine or a non-uniform grid is needed.

In the second part, this approach was applied to analyze the effect of thermoradiative parameters such as conduction-radiation parameter, scattering albedo and optical thickness on the temperature and heat flux distributions for three-dimensional J-shaped enclosure. It is evidenced from the results that the magnitude of the ratio of the radiation heat transfer to the conduction heat transfer should be at least unity to require a radiation heat transfer calculation $\left(N_{cr} \leq 1\right)$. Also, it seems that the dimensionless total heat flux is insensitive to the scattering albedo ω.

The proposed model leads to satisfactory solutions with comparison to reference data. Therefore, this method is strongly recommended and its formulation with the blocked-off-region procedure develops an accurate and simple numerical tool to deal with 3D radiative heat transfer in complex geometries, especially in the cases in which radiation is coupled with other heat transfer modes or CFD problems. Finally, this approach can be extended to anisotropic diffusion cases or to non-gray media. These problems remain for future investigation.

NOMENCLATURE

E	emissive power [$\mathrm{W/m^2}$]	θ	dimensionless temperature
G	incident radiation [$\mathrm{W/m^2}$]	κ	absorption coefficient [$\mathrm{m^{-1}}$]
I	radiation intensity [$\mathrm{W/m^2.sr}$]	ρ	reflectivity of the surface

$\bar{I}$	dimensionless radiation intensity
k	thermal conductivity [$\mathrm{W/m.K}$]
L	length [m]
M	number of discrete directions
n	unit vector normal to the surface
N_{cr}	conduction-radiation parameter
q	heat flux [$\mathrm{W/m^2}$]
Q	dimensionless heat flux
r	position vector [m]
S	source term [$\mathrm{W/m^3}$]
T	temperature [K]
w	weight of angular quadrature
x, y, z	coordinate [m]
X, Y, Z	dimensionless coordinate

Greek symbols

β	extinction coefficient [$\mathrm{m^{-1}}$]
ε	emissivity of a surface
σ	Stefan-Boltzmann constant $= 5.670\times10^{-8}\ \mathrm{W/m^2.K^4}$
σ_s	scattering coefficient [$\mathrm{m^{-1}}$]
τ	optical thickness
Φ	scattering phase function
ω	scattering albedo
Ω	direction vector

Subscripts

*	reference quantity
b	black body
m	discrete direction
p	nodal point
r	radiation
x, y, z	coordinate

Superscripts

$'$	incoming direction
n	iteration step

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