

THE USE OF FUZZY LOGIC THEORY FOR SELECTING APPROPRIATE TOOL STEELS WITH PRICE ANALYSIS*

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Abstract- Selection of tool steels is not limited to one type of steel for satisfying customer needs. On the other hand, technical specifications of tool steels are not absolute and it consistently deviates from the mean. Thus in this paper, the fuzzy logic theory has been used for selecting appropriate tool steels with price analysis. The main steps are as follows:

1. Defining and obtaining four technical features and prices of tool steels manufactured in Iran.
2. Calculating the mean and standard deviation of some technical features.
3. Defining and establishing relevant membership functions.
4. Relating tool steels to the amount of their favourability for customers.
5. Determining customer's desirable tool steels by using fuzzy logic theory.

In this paper, the possibility of reducing tool steels variety is also provided, leading to a decrease in the total cost of tool steels for the manufacturer.

Keywords- Tool steels selection, fuzzy logic, customer's needs, price analysis

1. INTRODUCTION

Fuzzy logic is used in uncertain environments so that experiments done by experts can be converted into mathematical languages. It is also possible to use it for selecting appropriate tool steels for customer needs. On the basis of fuzzy logic, it is possible to maximize the utilization of customer satisfaction.

We propose the fuzzy logic method because this method has not been used for local tool steels and has a real concept of natural actuation, which is something people face everyday. This logic seems to be a natural phenomena in a real form. It means that this fuzzy logic method shows that the satisfactory value for tool steels is a continuous value between zero and one.

Availability, technical specifications, hardness depth, and price are effective parameters for selecting appropriate tool steels. Technical specifications and the price of tool steels manufactured in Iran are obtained from manufacturers. The average and standard deviation of some technical specifications are then calculated and recorded according to Table 1 [1]. These parameters are shown as an expression form or a range of quantity defined by customer i . After studying the nature of selecting appropriate tool steels for customer i , one descriptive variable (toughness ($k = 1$)) and three numerical variables (untreated hardness ($k = 2$), hardness after heat treatment ($k = 3$), and maximum working temperature ($k = 4$)) are used for selecting appropriate tool steels [2-4].

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Table 1. Price and technical specification of tool steels manufactured in Iran *

No.	Steel	C %	$\bar{X}_{(j,2)}$ HB**	$\bar{X}_{(j,4)}$ 0K ****	Hardness after heat treatment $\bar{X}_{(j,3)}RC$ ***								Price ***** (Rial/kg)	
					1	2	3	4	5	6	7	8		
1	IASC2067	0.95-1.1	230	453	64	63	60							
2	IASC2080	1.9-2.2	250	523	63	62	60	58						
3	IASC2210	1.1-1.25	220	523	64	61	58							
4	IASC2379	1.5-1.6	250	523	64	63	60	58						20475
5	IASC2436	2-2.25	250	523	64	63	60	58						19200
6	IASC2510	0.9-1.05	215	523	64	62	57	53						20000
7	IASC2542	0.4-0.5	225	573	57	56	54	52						21000
8	IASC2550	0.55-0.65	225	573	60	59	56	52	47					18500
9	IASC2842	0.85-0.95	220	523	64	62	58							11000
10	IASC2601	1.55-1.75	250	523	63	61	59	58						
11	IASC2721	0.45-0.55	250	573	59	56	52	48	44					
12	IASC2767	0.4-0.5	260	523	56	54	51	48	42					
13	IASC2343	0.36-0.42	235	923	54	55.6	53.4	48.5	39.6	32				24600
14	IASC2344	0.37-0.43	235	923	54.8	54	55.6	54	50.2	42.2	37			21200
15	IASC2365	0.28-0.35	230	943	50.2	49.4	48.5	47.6	42.2	34				
16	IASC2367	0.35-0.4	235	973	55.6	52.7	45.5	37						20475
17	IASC2567	0.25-0.35	240	953	51.2	49.4	49.8	50.2	49.4	46.7	35.5			
18	IASC2713	0.5-0.6	240	923	52.7	48.5	46.7	44.4	42.2	39.6	34	27.5		24000
19	IASC2714	0.5-0.6	250	923	55.6	51.2	49.4	47.6	44.4	42.2	38.4			15140
20	IASC1545	1-1.1	190	573	65									
21	IASC1645	1-1.1	190	573	65									
22	IASC1730	0.4-0.5	190	573	58									18000
23	IASC1740	0.55-0.65	207	573	58									18500
24	IASC3207	1.2-1.35	270	843	65									
25	IASC3255	0.75-0.83	270	853	64									
26	IASC3343	0.86-0.94	252	833	64									
27	IW100	0.25-0.35	240	973	51.2	49.5	50.2	51.2	49.4	45.5	35.5			
28	IS300	0.72-0.8	270	853	64									
29	IS706	1.05-1.15	270	843	64									
30	IK305	0.9-1.05	230	673	63	62	60	57						20600
31	1.2313	0.16-0.23	200	923	49.4	44.4	43.3	40.7	38.4	35.5	32	27.5		

*Technical data obtained from *key to steel* [1].

** $S_{(j,2)}HB$ Standard deviation: 5

*** $S_{(j,3)}RC$ Standard deviation: 0.5

**** $S_{(j,4)}^0K$ Standard deviation: 5

***** Shape of tool steel: Round bar, Weight: 10 tons, 1 US \$ = 8000 Rials (Iranian currency).

Membership functions for the above technical features have been defined by normal distribution curve and LR function. In addition, after considering the nature of selecting appropriate tool steels, conjunction, and disjunction fuzzy rules are used. Also, satisfactory tool steels for customer *i* are graded by price cheapness, then combined with graded technical specifications ($F_{(i,j)}$). Thus, the number is generated in order to show us the final value of customer *i* satisfaction for tool steel *j* ($F_{f(i,j)}$). So, appropriate tool steels for customer(s) can be selected by fuzzy conjunction and disjunction rules. With this logic, the value of satisfaction for tool steel *j* appears as a continuous value between zero and one. Zero means that tool steel *j* cannot be supplied customer *i*. One means that tool steel *j* can completely supplied customer *i*. Thus, the number between zero and one can assign to customer *i* as a quantity or a

certain value. Also, sensitivity analysis of each criterion is done by a questionnaire from an expert customer.

Wang and Chang [5] proposed a fuzzy multi-criteria decision-making approach to select tool steel materials with fuzzy environments. The criteria are presented in fuzzy linguistic values and fuzzy numbers. Arithmetic operations have been used for ranking fuzzy numbers. Chen [6] has proposed a new method for selecting steel materials by the use of simple arithmetic operations.

2. MEMBERSHIP FUNCTIONS

a) Definition of toughness membership function

It is possible to evaluate customer needs in terms of properties such as toughness by different methods. However, factors such as toughness have a limited range of values depending on carbon equivalent changes in the limited range. The toughness is used for selecting and satisfying customer needs.

The toughness descriptive variable ($k = 1$) is divided into five sets: very tough, tough, a little tough, brittle, and very brittle. It is obvious that if the amount of carbon in steel is increased, toughness is decreased and hardness is increased. If the amount of Cr is also increased, especially greater than 10%, then toughness is strongly decreased and hardness is strongly increased. Nickel and magnesium are also effective in increasing toughness by small limited values. For this reason, and according to the values of the first hardness after the heat treatment listed in Case 1 of Table 1, the following suppositions are granted:

- Tool steel with 49.4 RC hardness is completely "very tough"
- Tool steel with 54 RC hardness is completely "tough"
- Tool steel with 60 RC hardness is completely "little tough"
- Tool steel with 64 RC hardness is completely "brittle"
- Tool steel with 65 RC hardness is completely "very brittle"

Primary membership functions in Table 2 are defined by fuzzy concepts [7-9]. By considering statistical premises [9] and toughness nature, and also using the LR function recorded in Table 3 [8, 10-13], modified membership functions are calculated and recorded in Table 4. Also, the limitation of modified membership functions is obtained from Table 4 and recorded in Table 5. By considering the statistical premise [9] and data in Table 1, the technical membership degree is calculated for customer i by a defuzzification method because the carbon percent is not absolute and it is in a limited range [8, 10-13]. It is done as follows:

$$F_{(i,j,l)} = \frac{(0 \times N_0) + (1 \times N_1) + (N \times 1)}{(N_0 + N_1 + 1)} \quad (1)$$

$$N_0 = \frac{D_0}{0.01} \quad (2)$$

N_0 = Number of ZERO degree repeated for each carbon step

D_0 = Difference between the greatest and smallest carbon contents having ZERO degree

$$N_1 = \frac{D_1}{0.01} \quad (3)$$

N_1 = Number of ONE degree repeated for each carbon step.

D_1 = Difference between the greatest and smallest carbon contents having ONE degree.

$$N = \frac{\int_S^G L.R.function}{G - S} \tag{4}$$

N = LR function average between S and G carbon content

G = Greatest carbon content having not ZERO or ONE degree

S = Smallest carbon content having not ZERO or ONE degree

When a customer requests tool steel for "little tough", it means that very tough tool steel, tough tool steel, and little tough tool steel can be supplied according to her/his request. Therefore, more toughness is required for customer satisfaction by selecting appropriate tool steel. Also, the toughness membership degree is determined by a fuzzy disjunction rule.

Table 2. Primary membership function for toughness, (the first hardness in Table 1 Case 1)

Data	Set name				
	Very tough	Tough	Little tough	Brittle	Very brittle
$hardness \leq 49.4$	1				
$hardness \geq 49.4$	$\frac{65 - hardness}{65 - 49.4}$				
$hardness \leq 54$		$\frac{hardness - 49.4}{54 - 49.4}$			
$hardness \geq 54$		$\frac{65 - hardness}{65 - 54}$			
$hardness \leq 60$			$\frac{hardness - 49.4}{60 - 49.4}$		
$hardness \geq 60$			$\frac{65 - hardness}{65 - 60}$		
$hardness \leq 64$				$\frac{hardness - 49.4}{64 - 49.4}$	
$hardness \geq 64$				$\frac{65 - hardness}{65 - 60}$	
$hardness \leq 65$					$\frac{hardness - 49.4}{65 - 49.4}$
$hardness \geq 65$					1

Table 3. LR function

Data	Set name				
	Very tough	Tough	Little tough	Brittle	Very brittle
$x \notin [sl \quad su]$	0				
$sl \leq x \leq a$	$0.5 \times \left(\frac{(x - sl)}{(a - sl)} \right)^e$				
$a \leq x \leq b$	$1 - 0.5 \times \left(\frac{(b - x)}{(b - a)} \right)^e$				
$b \leq x \leq c$	1				
$c \leq x \leq d$	$1 - 0.5 \times \left(\frac{(x - c)}{(d - c)} \right)^e$				
$d \leq x \leq su$	$0.5 \times \left(\frac{(su - x)}{(su - d)} \right)^e$				

Table 4. Modified membership function for toughness*

Data **	Set name				
	Very Tough	Tough	Little Tough	Brittle	Very Brittle
$x \notin [sl \quad su]$	0	0	0	0	0
$sl \leq x \leq a$	1	$553.8(x - 0.3)^{2.34}$	$109.4(x - 0.4)^{2.34}$	$64.9(x - 0.75)^{2.34}$	$29.53(x - 1.15)^{2.34}$
$a \leq x \leq b$	1	$1 - 553.8(0.4 - x)^{2.34}$	$1 - 109.4(0.6 - x)^{2.34}$	$1 - 64.9(1 - x)^{2.34}$	$1 - 29.53(1.5 - x)^{2.34}$
$b \leq x \leq c$	1	1	1	1	1
$c \leq x \leq d$	$1 - 109.4(x - 0.4)^{2.34}$	$1 - 553.8(x - 0.6)^{2.34}$	$1 - 109.4(x - 1)^{2.34}$	$1 - 64.9(x - 1.5)^{2.34}$	1
$d \leq x \leq su$	$109.4(0.6 - x)^{2.34}$	$553.8(0.7 - x)^{2.34}$	$109.4(1.2 - x)^{2.34}$	$64.9(1.75 - x)^{2.34}$	1

* e = 2.34 according to Table 3.

** x is carbon in wt %.

Table 5. Limitation of the modified membership function for toughness

Data	Set name				
	Very tough	Tough	Little tough	Brittle	Very brittle
sl	0	0.3	0.4	0.75	1.15
a	0	0.35	0.5	0.875	1.325
b	0	0.4	0.6	1	1.5
c	0.4	0.6	1	1.5	2.3
d	0.5	0.65	1.1	1.625	2.3
$\bar{x}_{(j,4)}$	0.6	0.7	1.2	1.75	2.3

b) Definition of untreated hardness function and hardness after heat treatment function

Untreated hardness membership function and hardness after heat treatment membership functions are defined by a mathematical probability rule as follows [9]:

$$F_{(i,j,k)} = \text{prob}(m_{(i,k)} \leq X_{(j,k)} \leq M_{(i,k)}), \quad i=1 \text{ to } I=3 ; j=1 \text{ to } J=31 ; k=2, 3 \quad (5)$$

This shows how much the specification k of steel j can satisfy customer i.

c) Definition of maximum working temperature

The maximum working temperature membership function is defined by a mathematical probability rule, however there is a little difference. When maximum working temperature is less than $\bar{X}_{(j,4)}$, it can be completely supplied to customer i based on the request. When it is greater than $\bar{X}_{(j,4)}$, it can be supplied to customer i based on a certain quantity. This function is defined as follows:

$$F_{(i,j,k)} = \text{prob}(M_{(i,k)} \leq X_{(j,k)}), \quad i=1 \text{ to } I=3 ; j=1 \text{ to } J=31 ; k=4 \quad (6)$$

d) Combination of technical specifications

Lack of technical specification is not acceptable, therefore the minimum fuzzy rule helps us for determining technical specifications satisfaction degree. The combination of membership functions of the satisfaction degree of technical specifications is defined as follows [6-10]:

$$F_{(i,j)} = \min(F_{(i,j,k)}), \quad i=1 \text{ to } I=3 ; j=1 \text{ to } J=31 ; k=1 \text{ to } 4 \quad (7)$$

e) Reliability

Technical specifications reliability (a_i) of tool steel j is determined by customer i . In this case, $F_{(i,j)} \geq a_i$. If $F_{(i,j)}$ (minimum membership degrees of technical specifications for tool steel j and customer i) is smaller than a_i , as listed in No. 5 of Table 6, then it is not acceptable for customer i .

Note: If tool steel can not be supplied to customer i (it means that $F_{(i,j)} < a_i$ or $F_{(i,j)} = 0$),
Then it is not approved.

F) Price membership function

If the price is cheaper, then the customer is better satisfied. Price membership function or price cheapness degree function is a function showing the deviate value of the most expensive satisfaction tool steel j for customer i in a range limited form zero to one by fuzzy concepts as follows:

$$F_{P_{(i,j)}} = \frac{A - B}{C - D} \quad (8)$$

A = the most expensive satisfaction of tool steel j for customer i

B = price of satisfaction of tool steel j for customer i

C = the most expensive satisfaction of tool steel j for customer i

D = the cheapest satisfaction of tool steel j for customer i

g) Final membership function

Each customer (i) has a different importance degree for price and technical specifications. B_{1_i} and B_{2_i} are importance degrees of technical specifications and price cheapness for customer i according to Numbers 6 and 7 of Table 6. Final membership function or $F_{f_{(i,j)}}$ is defined for the satisfaction of tool steel j and customer i as follows:

$$\text{if } F_{(i,j)} = 0 \quad \text{Then } F_{f_{(i,j)}} = 0 \quad (9)$$

$$\text{if } F_{(i,j)} \neq 0 \quad \text{Then } F_{f_{(i,j)}} = (B_{1_i} \times F_{(i,j)}) + (B_{2_i} \times F_{P_{(i,j)}})$$

$$\text{also } B_{1_i} + B_{2_i} = 1 \quad \text{and} \quad B_{1_i} > B_{2_i}$$

3. PRICE ANALYSIS FOR CUSTOMERS

In this paper, a questionnaire is designed and three expert customers have replied back. A sensitivity analysis of each criterion is then carried out. Tool steel is selected for three customers as listed in Table 6. These customers have individual requests. Satisfaction of tool steels is graded for customers by Equations 5 to 9. Satisfaction of tool steel is specified for customers 1, 2 and 3 by the fuzzy conjunction rule, which is calculated and recorded in Column 8 of Table 7 (F_{f_j}). It is done as follows:

$$F_{f_j} = \min (F_{f_{(i,j)}}) \quad (10)$$

Finally, IASC2367 is selected as the greatest satisfaction tool steel for all three mentioned customers by a fuzzy disjunction rule and recorded in Row 9 of Table 7 (F_f). It is done as follows:

$$F_f = \max (F_{f_j}) \quad (11)$$

Satisfaction of tool steels is listed in Column 8 of Table 7 for all three mentioned customers. If the price of IASC2367 steel is increased by 18%, 20%, and 44% (e.g., better control or offer and demand) then tool steels with price cheapness are calculated according to Equation 8 for all three customers. Also,

final membership degree for appropriate tool steel j and customer i ($F_{f(i,j)}$) is calculated according to Equation 9 for all three customers. $F_{f(i,j)}$, F_{f_j} and F_f are shown in Figure 1. Also, the most satisfied tool steel is IASC2367 for all customers. After increasing the price to 20%, and 44%, the most satisfied tool steel changes to IASC2343 for all customers.

Table 6. Request steel for customers 1, 2, and 3

No	Variable Data	unit	Customer 1	Customer 2	Customer 3
1	Hardness	HB	170 - 250	200-250	170-260
2	Hardness after heat treatment	RC	52-60	51-67	50-58
3	Max. working temperature	$^{\circ}C$	50	250	600
4	Toughness	---	tough	little tough	very tough
5	Reliability	%	99	98	99
6	Technical importance	---	0.5	0.8	0.8
7	Price importance	---	0.5	0.2	0.2

Table 7. Satisfaction of tool steel j ($F_{f(i,j)}$) calculated for customer 1, 2, and 3 and the most satisfied tool steel for all customers (F_{f_j})

No.	j from Table 1	Steel	Case	$F_{f(1,j)}$	$F_{f(2,j)}$	$F_{f(3,j)}$	F_{f_j}
1	13	IASC2343	1	0.5477	0.8834	0.971	0.5477
2	13	IASC2343	2	0.5477	0.8834	0.971	0.5477
3	13	IASC2343	3	0.5477	0.8834	0.971	0.5477
4	14	IASC2344	1	0.5	0.8686	0.9	0.5
5	14	IASC2344	2	0.5	0.8686	0.9	0.5
6	14	IASC2344	3	0.5	0.8686	0.9	0.5
7	14	IASC2344	4	0.5	0.8686	0.9	0.5
8	16	IASC2367	1	0.808	0.9562	1	0.808
9	F_f						0.808

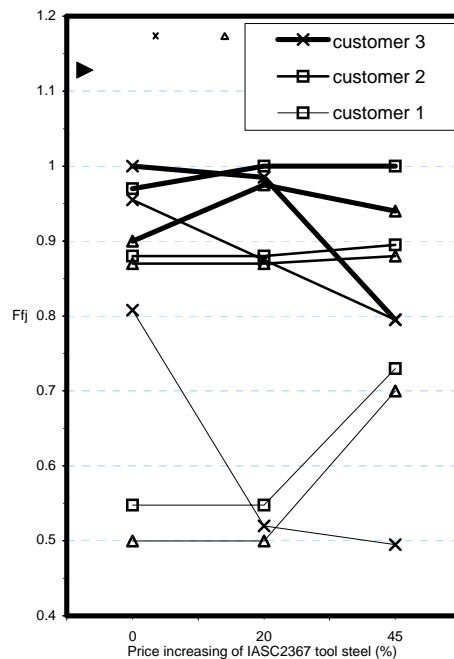


Fig. 1. Satisfaction tool steel j for all customers (F_{f_j}) variation with an increase of IASC2367 price

4. RESULTS AND DISCUSSION

The computational results are summarized as follows:

- 1) As shown in Table 7, some membership degrees are equal. Why is it so? $F_{(i,j)}$ may be equal for customer i and all heat treatment cases of one tool steel (according to Equations 5 to 7 and Table 4). However, tool steel price is certainly different for a number of heat treatment cases of one tool steel. Thus, $F_{p_{(i,j)}}$ is certainly different for alternative heat treatment cases of one tool steel according to Eq. (8). Then, $F_{f_{(i,j)}}$ will be different for each tool steel according to Equation 9. However, heat treatment cost is ignored due to the fact that tool steel price is constant for all heat treatment cases of one tool steel. Thus, $F_{f_{(i,j)}}$ and F_{f_j} may be equal for some membership degrees.
- 2) Increasing the price of the IASC2367 Case 1 for customer 1 is more effective than other customers since price is more important for customer 1 as compared with customer 2 and 3 according to Number 7 of Table 6 and Eq. (9). In other words, $B_{2_1} = 0.5$ is greater than $B_{2_2} = 0.2$ and $B_{2_3} = 0.2$ as shown in Fig. 1.
- 3) Customer satisfaction was supplied according to Section 2-5 by a_i .
- 4) The decrease in a variety of tool steels is resulted in a decrease in the total cost of tool steels for the manufacturers. Because only one kind of tool steel (IASC2367 case1) was selected for three customers, in which all customer requests were supplied according to Section 3.
- 5) Occasionally, tool steels are the same satisfaction for one individual application for this reason, *key to steel* is used, because each tool steel is more satisfactory than others for one individual application. Thus, the selection of the appropriate tool steel was formulated and tool steel satisfaction was graded from zero to one by this method as shown in Table 7.
- 6) If the price of selected tool steels increases, then F_{f_j} changes according to B_{2_i} and it increases according to Eqs. (8) and (9). Thus, the most satisfactory tool steel may be changed, as shown in Figure 1. When the price of IASC2367 Case 1 raises (e.g., 44 %) the most satisfactory tool steels will be IASC2343 Cases 1, 2, and 3 instead of IASC2367 Case 1.
- 7) Selecting appropriate tool steel is analyzed by the price for customers according to Section 4.

NOMENCLATURE

i	customer number
I	customer quantity
j	tool steel number
J	tool steel quantity
k	technical specification number
$\bar{X}_{(j,k)}$	average of technical specification k for tool steel j
$S_{(j,k)}$	standard deviation of technical specification k for tool steel j
$m_{(i,k)}$	minimum allowed technical specification k for tool steel j
$M_{(i,k)}$	maximum allowed technical specification k for tool steel j
<i>Prob.</i>	probability rule
<i>Max</i>	fuzzy disjunction rule
<i>Min</i>	fuzzy conjunction rule
N	L.R. function average between S and G carbon content
G	greatest carbon content which does not have ZERO or ONE degree
S	smallest carbon content which does not have ZERO or ONE degree
$F_{(i,j,k)}$	membership degree of tool steel j for technical specified k and customer i satisfied

$F_{(i,j)}$	minimum membership degrees of technical specifications for tool steel j and customer i
a_i	reliability percent of technical specifications for customer i satisfied
$F_{p(i,j)}$	membership degree of price cheapness for appropriate tool steel j and customer i
B_{1_i}	importance degree of technical specifications for customer i
B_{2_i}	importance degree of price cheapness for customer i
$F_{f(i,j)}$	final membership degree for appropriate tool steel j and customer i
F_{f_j}	minimum of $F_{f(i,j)}$ for satisfactory tool steel j for all customers
F_f	maximum of F_{f_j} for selecting the most satisfactory tool steels for all customers

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