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MODELING OF FROST GROWTH ON A HORIZONTAL CIRCULAR CYLINDER UNDER NATURAL CONVECTION USING FRACTAL GEOMETRY ANALYSIS*

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Abstract– In the present study fractal geometry is used to model frost growth around a cooled horizontal circular cylinder having constant surface temperature. Fractal geometries are very applicable in this area because phase changes such as melting and solidifications are simulated by conventional methods but frost formation is a most complicated coupled heat and mass transfer and conventional mathematical functions cannot capture the effects of all parameters on its growth and development because this process is influenced by many factors and it is a time dependent process. Data for fractal analysis and modeling are collected from the experimental measurements. In the present work a fractal geometry based on the Von Koch curve idea is used to model frost growth procedure, especially in frost thickness and density. Comparison is performed between fractal modeling and experimental measurements. Results show that fractal geometry is capable to model frost formation process over a cold horizontal cylinder. According to the obtained results and based on the developed model frost formation over cylinders with different diameters are modeled.

Keywords– Frost growth, fractal geometry, Von-Koch curves, natural convection, horizontal cylinder

1. INTRODUCTION

Frost formation on cylinders is of considerable practical interest in the efficient design of heat pumps and refrigerators. There is a growing demand for a better understanding of frost phenomenon in applications such as heat exchangers, air cooling for air conditioning system, etc.

Phase changes such as melting and solidification are simulated by conventional methods [1], but frost formation is a complicated coupled heat and mass transfer and conventional mathematical functions cannot capture the effects of all parameters on its growth and development because this process is influenced by many factors and it is a time dependent process.

To prevail over these difficulties and for further detailed analysis, new approaches and methods such as fractal geometry, artificial neural networks, etc. can be used. These approaches reduced the cost of studies and saving computational time. As mentioned, fractal geometry can be an alternative and a new attempt in this area. Fractals are an integral part of the natural world since most of the naturally formed geometries are fractals. For example clouds, coastlines, lightning, snow flakes, cauliflower and even the system of our blood vessels are all fractal in nature [2–5]. In fact, anything which is fragmented or irregular can be a fractal. Fractals are irregular and they cannot be described by Euclidean geometries.

They are also characterized by self-similarity which essentially means that each part of the geometry is a reduced scale image of the whole [2]. Even when the geometry is not identical at all the magnification levels, it can be fractal if there is any property of the geometry that repeats itself as it is magnified. This type of self-similarity is called statistical self-similarity [6].

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The theory of fractals aids in analyzing the response of practical systems besides examining the complex geometries of nature. Fractal theory has a wide variety of applications ranging from economy [7], environmental sciences [8], mathematics [9, 10], neurology [11, 12] and thermal sciences [13, 14]. Fractals have been used by various researchers for modeling and predictions in the field of engineering systems such as crack growth, fracture, roughness, and technological devices, etc. Also, fractals are used to analyze frost, crystals, and material structures [15-18]. But from a review of the literature, it can be observed that fractals have rarely been employed for modeling frost formation procedure.

In the present work, fractals are employed to model frost formation over a cold horizontal cylinder under natural convection and comparison is made with experimental results. Measurements of Chen and Oosthuizen [19], Yaghoubi and Khoshnazar [20], and Tahavvor and Yaghoubi [21, 22] are used to generate the database for fractals, and new results of frost growth over horizontal cylinder are developed.

2. ACCUMULATED DATA

For modeling frost growth, fractal geometry has been designed according to the experimental results of Tahavvor and Yaghoubi [21, 22], Chen and Oosthuizen [19], and Yaghoubi and Khoshnazar [20]. In all experiments, results were obtained with varying environmental condition and cylinder surface temperature. In Ref. [19] two tube diameters are used and in other works only one tube diameter was used. The experiments condition of Ref. [19] for pipes of large diameter and small diameter are listed in Table 1. Tables 2 and 3 show the test conditions of experiments of Ref. [20] and Ref. [21], respectively.

Table 1. Experimental data of [3]

Cylinder diameter = 22.4 cm			
Test Number	T_w (ºC)	T_a (ºC)	RH (%)
I	-2.66	21.28	47.20
II	-11.75	25.28	48.18
III	-6.54	25.71	49.01
IV	-2.58	24.96	53.00
V	-6.67	25.81	55.89
VI	-3.38	26.53	53.00
Cylinder diameter = 15.8 cm			
Test Number	T_w (ºC)	T_a (ºC)	RH (%)
VII	-6.18	28.90	47.90
VIII	-8.39	28.19	42.60
IX	-5.96	29.13	50.10
X	-2.51	30.34	54.60
XI	-5.19	30.34	51.10

Table 2. Experimental data of [4]

Cylinder diameter = 8.00 cm			
Test Number	T_w (ºC)	T_a (ºC)	ω (kg kg_{DA}-1)
XII	-4.00	18.00	0.0064
XIII	-8.00	22.00	0.0089

Table 3. Experimental data of [5]

Cylinder diameter = 7.7 cm			
Test Number	T_w (ºC)	T_a (ºC)	RH (%)
1	-5.21	30.0	30.0
2	-4.50	30.0	50.0
3	-7.34	30.0	37.0
4	-9.30	25.0	37.0
5	-10.07	20.0	37.0

3. FRACTALS

Fractal is a mathematical set that has a fractal dimension that usually exceeds its topological dimension and may fall between the integers. Fractals are typically self-similar patterns, where self-similar means they are "the same from near as from far". Fractals may be exactly the same at every scale, or may be nearly the same at different scales. The definition of fractal goes beyond self-similarity to exclude trivial self-similarity and include the idea of a detailed pattern repeating itself.
The mathematics behind fractals began to take shape in the 17th century but one of the milestones came in 1904 when Helge Von Koch, extending the ideas of Poincaré and dissatisfied with Weierstrass's abstract and analytic definition, gave a more geometric definition including hand drawn images of a similar function, which is now called the Von Koch curve.

Finally, Mandelbrot solidified hundreds of years of thought and mathematical development in coining the word "fractal" and illustrated his mathematical definition with striking computer-constructed visualizations [23, 24]. The Von Koch fractal curves and islands are perhaps the most beautiful fractals, as illustrated in Fig 1.

a) Fractal geometry

Hayashi et al. [25] and Tao et al. [26, 27] observed that frost formation under forced convection condition consists of two periods. During the first period frost can be simulated as a forest of ice columns until a transition time, after which the frost may be considered as a homogenous porous medium, as schematically illustrated in Fig. 2 appears to have been taken from R.O. Piucco et al. [28].

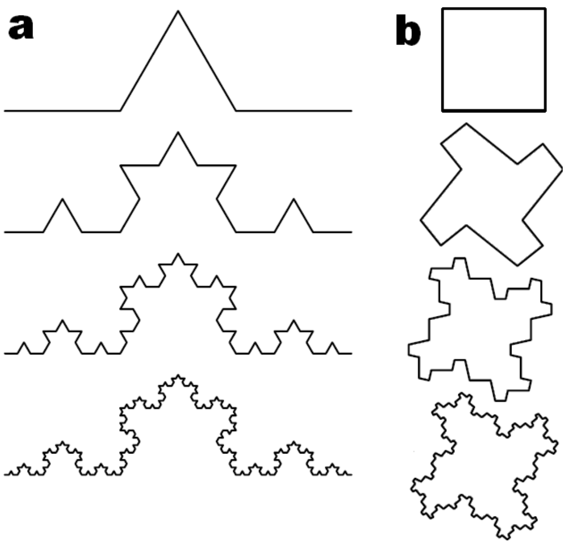

Fig. 1. Von Koch fractal (a) curves and (b) islands [8]

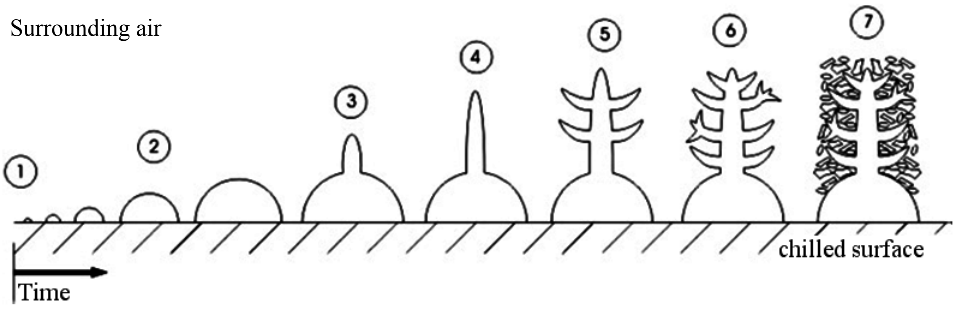

Fig. 2. Frost growth procedure [9]

Also, same stages are observed and modeled by Tahavvor and Yaghoubi under natural convection condition [22]. According to these observations and also according to von Koch fractal curve idea, the segment shown in Fig. 3a is used as an initial segment of frost crystal. The dimensions of fractal segments (showed in Fig. 3a) vary with time. This variation can be considered as a function of time and ambient temperature. Frost models suggest that cold surface temperature has no appreciable effect on initial fractal segment dimensions. The relationships between fractal segment dimensions, time, cold surface temperature, and relative humidity are determined as follows:

$$a(t,T_w,RH)=\frac{C_a}{50}\begin{pmatrix} p_{00}+p_{10}T_w+p_{01}t+p_{20}T_w{}^2+p_{11}T_wt+p_{02}t^2+p_{21}T_w{}^2t+ \\ p_{12}T_wt^2+p_{03}t^3+p_{22}T_w{}^2t^2+p_{13}T_wt^3+p_{04}T_w{}^4 \end{pmatrix} \tag{1}$$

$$b(t,T_w,RH)=\frac{C_b}{50}\begin{pmatrix} p_{00}+p_{10}T_w+p_{01}t+p_{20}T_w{}^2+p_{11}T_wt+p_{02}t^2+p_{21}T_w{}^2t+ \\ p_{12}T_wt^2+p_{03}t^3+p_{22}T_w{}^2t^2+p_{13}T_wt^3+p_{04}T_w{}^4 \end{pmatrix} \tag{2}$$

$$L(T_w)=\frac{1}{50}\left(p_{01}T_w{}^2+p_{02}T_w+p_{03}\right) \tag{3}$$

$$h={}^7\!/_5\equiv cte \tag{4}$$

Where C is determined from the following relation:

$$C_{a,b}(RH)=p_{01}+p_{02}\frac{\ln RH}{RH} \tag{5}$$

Values of coefficients of Eqs. (1-3) and 5 are listed in Table 4.

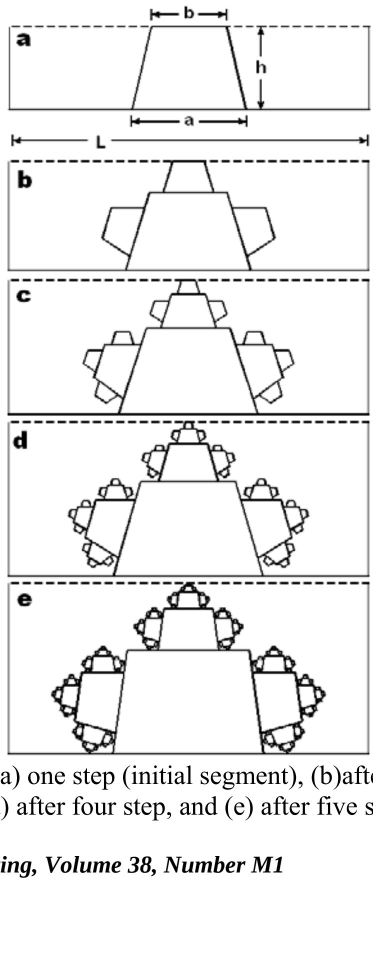

Fig. 3. Fractal segment after (a) one step (initial segment), (b)after two step, (c)after three step, (d) after four step, and (e) after five step

Table 4. Coefficients of Eqs. (1), (2), and (3)

	Equation (1)	Equation (2)	Equation (3)	C_a	C_b
p_{00}	203.8	192.6	-	-	
p_{10}	49.26	36.33	-	-	
p_{01}	0.1313	-0.1342	7.853	2.014	2.342
p_{20}	2.922	2.082	-	-	
p_{11}	-0.127	-0.04128	-	-	
p_{02}	-0.001	0.00013	96.91	-10.314	-13.694
p_{21}	-0.01013	-0.004213	-	-	
p_{12}	7.319e-5	-7.816e-6	-	-	
p_{03}	1.233e-6	-2.778e-7	795.2	-	
p_{22}	7.338e-6	3.587e-6	-	-	
p_{13}	6.677e-9	3.784e-8	-	-	
p_{04}	-4.601e-10	2.604e-10	-	-	

In modeling process each edge of segment stands as a base of a new segment. This process is then repeated. Figures 3b through 3e represent the fractal after two through five iterations respectively. Increasing the iteration number provides more detailed drawings; so, the length of the Von Koch curve increases at each step. In each step, the segments are divided and the number of segments is multiplied by four, hence length and height increase in each step.

4. RESULTS AND DISCUSSIONS

Fractal geometry is used to model frost thickness and density around a cooled horizontal cylinder under natural convection. Results for certain cases were compared with experimental data. The problem of frost formation on cold circular cylinder in natural convection condition is chosen to show the feasibility of the fractals to determine frost thickness and density for various conditions and provide new results for different conditions.

The results from experiments and modeling for RH = 37%, T_w = -8.51 °C, and T_a = 30 °C are used for validating the accuracy of the model. Results are compared in Fig. 4a and 4b. Very good agreement can be observed between results of fractals and experiments.

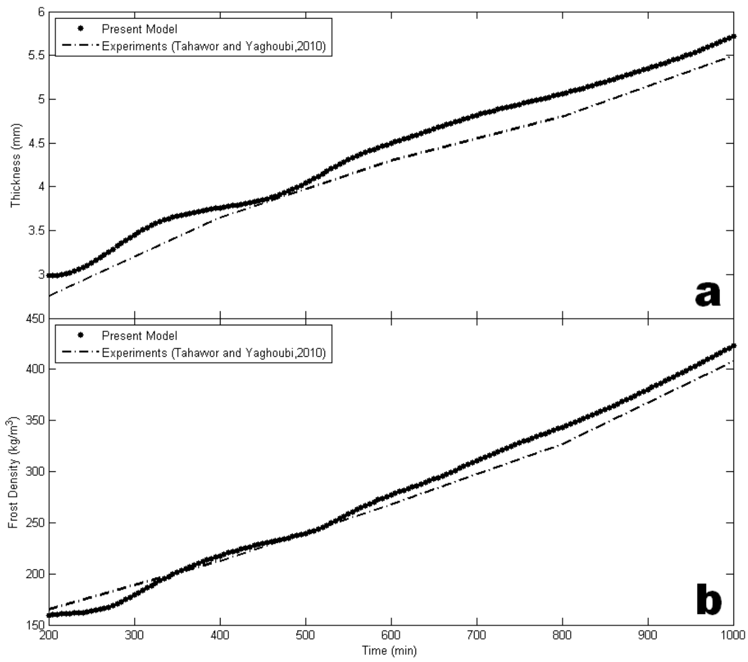

Fig. 4. Results of prediction of frost thickness and density for RH=37%, T_w=-8.51 °C, and T_a=30 °C

This model was also used to simulate frost formation under various conditions. Results of these tests are presented in Figs. 5 to 7. From these figures it is evident that fractal modeling is capable of accurately determining frost thickness and density from the input data.

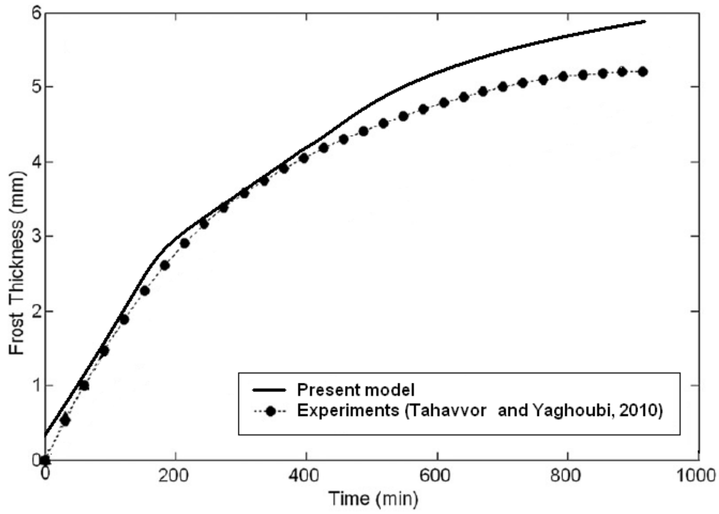

Fig. 5. Results of prediction of frost thickness for RH=50%, T_w=-4.5 °C, and T_a=30 °C

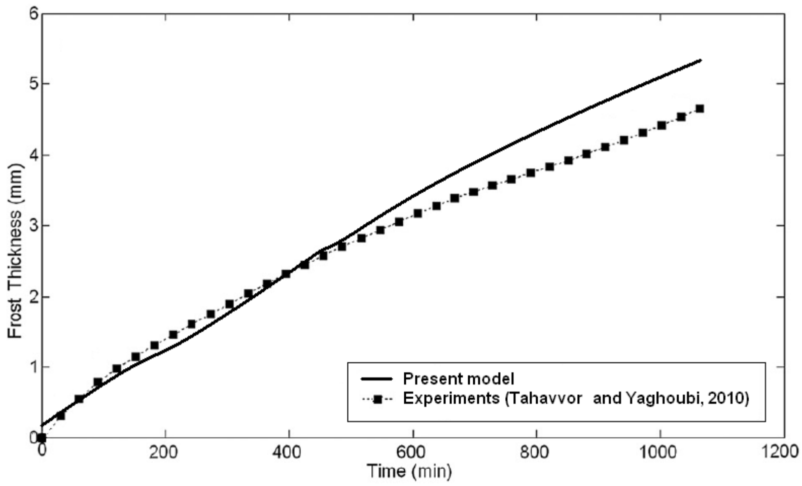

Fig. 6. Results of prediction of frost thickness for RH=30%, T_w=-5.21 °C, and T_a=30 °C

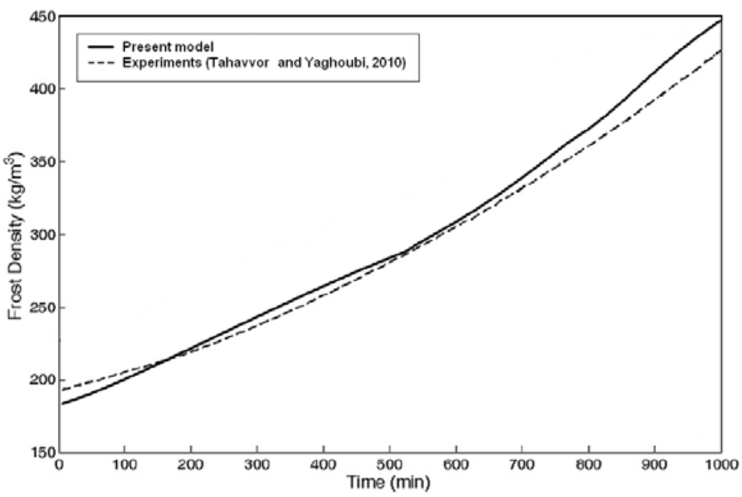

Fig. 7. Results of prediction of frost density for RH=30%, T_w=-5.21 °C, and T_a=30 °C

5. CONCLUSION

Fractal modeling is compared with experimental results for study of frost formation around a cold horizontal circular cylinder under natural convection for various cases.

- Von Koch fractal curves are capable of modeling frost growth.
- Very good agreement was found between fractals scheme and experimental results.
- Fractal method can easily be used to determine new results for frost thickness and frost density over a horizontal cylinder in natural convection cooling with considerably less computational cost and time.
- Relative errors of fractal modeling relative to experimental measurements are less than 4% for frost density and 5% for frost thickness.

Several modelings show that cold surface temperature has no considerable effect on initial fractal segment dimensions.

NOMENCLATURE

a	large fractal segment base
b	small fractal segment base
h	fractal segment height
L	fractal site length
RH	relative humidity
t	time
T	temperature

Subscript

a	ambient
w	wall

REFERENCES

1. Solhjoo, S., Simchi, A. & Aashouri, H. (2012). Molecular dynamics simulation of melting, solidification and remelting processes of aluminum. *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, Vol. 36, No. M1, pp. 13-23.
2. Mandelbrot, B. B. (1977). *The fractal geometry of nature*. W. H. Freeman and Company, New York, 1977.
3. Gabrys, E., Rybaczuk, M. & Kedzia, A. (2006). Blood flow simulation through fractal models of circulatory system. *Chaos Solitons Fract*, Vol. 27, No. 1, pp 1–7.
4. Schneider, M. & Wohlke T. (1992). Simplified model for fractal dimension of clouds. *Physica A*, Vol. 189, Nos. 1–2, pp. 1-3.
5. Reiter, C. A. (2005). A local cellular model for snow crystal growth. *Chaos Solitons Fract*, Vol. 23, No. 4, pp. 1111–1119.
6. Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science*, Vol. 156, pp. 636–638.
7. Mulligan, R. F. & Koppl, R. (2011). Monetary policy regimes in macroeconomic data: an application of fractal analysis. *Quart. Rev. Econ. Finance*, Vol. 51, No. 2, pp. 201–211.
8. Yang, X. B., Jin, X. Q., Du, Z. M. & Zhu, Y. H. (2011). A novel model-based fault detection method for temperature sensor using fractal correlation dimension. *Build. Environ*., Vol. 46, No. 4, pp. 970–979.
9. Chen, J. & Ngai, S. M. (2010). Eigen-values and eigen-functions of one-dimensional fractal Laplacians defined by iterated function systems with overlaps. *J. Math. Anal. Appl*., Vol. 364, No. 1, pp. 222–241.
10. Pasic, M. & Tanaka, S. (2011). Fractal oscillations of self-adjoint and damped linear differential equations of second-order. *J. Appl. Math. Comput*., Vol. 218, No. 5, pp. 2281–2293.

11. King, R. D., Brown, B., Hwang, M. & Jeon, T. (2010). A. T. George and the Alzheimer's disease neuroimaging initiative, fractal dimension analysis of the corticalribbon in mild Alzheimer's disease. *Neuroimage*. Vol. 53, No. 2, pp. 471–479.
12. Iftekharuddin, K. M., Zheng, J., Islam, Md. A. & Ogg, R. J. (2009). Fractal-based brain tumor detection in multimodal MRI. *J. Appl. Math. Comput*., Vol. 207, No. 1, pp. 23–41.
13. Zhu, Q. Y., Xie, M. H., Yang, J. & Li, Y. (2010). A fractal model for the coupled heat and mass transfer in porous fibrous media. *Int. J. Heat Mass Tran*. Vol. 54, No. 7–8, pp. 1400–1409.
14. Xiao, B., Yu, B., Wang, Z. & Chen, L. (2009). A fractal model for heat transfer of nanofluids by convection in a pool. *Phys. Lett. A*., Vol. 373, No. 45, pp. 4178–4181.
15. Liu, Y., Liu, Z. & Huang, L. (2012). Experimental and theoretical investigations of the fractal characteristics of frost crystals during frost formation process. *Exp. Therm. Fluid Science*, Vol. 36, pp. 217-223.
16. Avery, I., Hall, F. R. & Sturgess, C. E. N. (1998). Fractal modelling of materials. *Journal of Materials Processing Technology*, Vol. 80–81, pp. 565-571.
17. He, K., Xu, C.Y., Zhen, L. & Shao, W. Z. (2008). Fractal growth of single-crystal α-Fe_2O_3: From dendritic micro-pines to hexagonal micro-snowflakes. *Materials Letters*, Vol. 62, No. 4–5, pp. 739-742.
18. Ta, D. & Marangoni, A. G. (2006). 3D fractal dimension of fat crystal networks. *Chemical Physics Letters*, Vol. 433, No. 1–3, pp. 248-252.
19. Chen, J. & Oosthuizen, P. H. (1994). Frost formation on a horizontal cylinder with free convection. M.Sc. thesis, Queen's University, Kingston, Canada.
20. Yaghoubi, M. & Khoshnazar, H. (2010). Experimental study of frost formation on a horizontal cylinder. *AIAA/ASME Joint Thermophysics and Heat Transfer Conference*, Chicago, USA.
21. Tahavvor, A. R. & Yaghoubi, M., Experimental and numerical study of frost formation by natural convection over a cold horizontal circular cylinder. *Int. J. Refrigeration*, Vol. 33, pp. 1444-1458.
22. Tahavvor, A. R. & Yaghoubi, M. (2009). Analysis of early-stage frost formation in natural convection over a horizontal cylinder. *Int. J. Refrigeration*, Vol. 32, pp. 1343-1349.
23. Mandelbrot, B. B. (1983). *The fractal geometry of nature*. Macmillan, New York.
24. Pesin, Y. & Climenhaga, V. (2009). *Lectures on fractal geometry and dynamical systems*. American Mathematical Society, Pennsylvania.
25. Hayashi, Y., Aoki, A., Adachi, S. & Hori, K. (1977). Study of frost properties correlating with frost formation types. *J. Heat Transf*., Vol. 99, pp. 239-245.
26. Tao, Y. X., Besant, R. W. & Mao, Y. (1993). Characteristics of frost growth on a flat plate during the early growth period. *ASHRAE Transactions*, Vol. 99, No. 1, pp. 746-753.
27. Tao, Y. X., Besant, R. W. & Rezkallah, K. S. (1993). A mathematical model for predicting the densification and growth of frost on a flat plate. *Int. J. Heat Mass Transf*., Vol. 36, pp. 353-363.
28. Piucco, R. O. (2008). A study of frost nucleation on flat surfaces. *Exp. Thermal Fluid Sci*., Vol. 32, pp. 1710-1715.