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EFFECTS OF CONTACT FORCE MODEL ON DYNAMICS CHARACTERISTICS OF MECHANICAL SYSTEM WITH REVOLUTE CLEARANCE JOINTS*

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Abstract– The contact force model is one of the important issues in dynamics analysis of mechanical systems with joint clearance. The main objective of this work is to present a computational study on the effects of the contact force models on dynamics characteristics of mechanical system with revolute clearance joints. The intra-joint contact forces that are generated at clearance joints are computed by considering several different elastic and dissipative approaches. A simple review of the constitutive laws utilized in this work is presented and analyzed. Finally, a well-known slider-crank mechanism with a revolute clearance joint is utilized to perform the investigation. The investigation results show that the dynamics characteristics of mechanical system with clearance are obviously shaking and the amplitude increases from the mechanism without clearance. The contact force model of the clearance joint has an important effect on the dynamics responses of mechanical system and the selection of appropriate contact force model of clearance joints plays a significant role in dynamics analysis of multibody mechanical system with revolute clearance joint.

Keywords– Clearance joint, contact force model, mechanical system, dynamic characteristics

1. INTRODUCTION

One of the important problems in dynamics of multibody mechanical system with revolute clearance joint is how to select the appropriate contact force model that best describes a given contact-impact event [1-4]. Clearances in mechanism are unavoidable due to assemblage, manufacturing errors and wear. The movement of the real mechanisms is deflected from the ideal mechanism and the motion accuracy is decreased due to joint clearances. These clearances modify the dynamic response of the system, justify the deviations between the numerical predictions and experimental measurements and eventually lead to important deviations between the projected performances of mechanisms and their real outcome.

Over the last few decades, effects of clearance on dynamics characteristics of mechanisms have been studied by many researchers [5-12]. All the researches indicate that contact and impact were the typical phenomena of mechanism with joint clearance. The contact-impact models of multibody system are mainly focused on the discrete analysis method and continuous contact analysis method [4, 13, 14]. The former is the use of coefficient of restitution and momentum balance at the time of contact. It assumes that the contact-impact is very short and does not change the overall configuration of the object. Then, the contact-impact process is divided into two stages, before and after impact, and relative sliding, viscous stagnation and reverse movement will occur between two objects after the impact. The latter assumes that interaction forces between the impact objects are continuous in the entire contact-impact process. This

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approach is in agreement with real contact-impact behavior of objects. The continuous contact force model is widely used for contact-impact analysis of mechanism with clearance and the elastic contact force is widely represented by Hertz contact law [15]. The continuous contact force model should include the energy dissipation of impact between the journal and bearing. There are various contact force models, mainly based on Hertz model, such as linear spring-damper model [16, 17], the nonlinear damping model of Hunt and Crossley [18], and the nonlinear model of Lankarani and Nikravesh [19]. More recently, Bai and Zhao [4] presented a new contact force model of revolute clearance joint, which was compared with the previous contact force models for contact and impact process analysis between the journal and bearing in clearance joint. However, the effects of the various contact force models on the dynamic responses of multibody system with clearance joint are less investigated. Therefore, this work concentrates on the comparison between various contact force models on the dynamics of multibody system with clearance joint.

Once the contact forces approach is adopted, the contact force model becomes the important factor influencing the precision of the simulation results. The contact force model of revolute joints with clearance is one of the important contents in dynamics analysis of mechanism with clearance. The numerical description of the collision phenomenon is strongly dependent on the contact force model used to represent the interaction between the joint components. Therefore, the constitutive contact force law utilized to describe contact-impact events plays a crucial role in predicting the dynamic response of mechanical systems and simulation of the engineering applications. Thus, this paper studies the influence of the use of various contact force models on the dynamic responses of multibody mechanical including dry revolute clearance joints. The planar slider-crank mechanism with revolute joint clearances is used as numerical example to demonstrate and validate the different contact force models presented in this work.

2. DEFINITION OF CLEARANCE

In general, a clearance joint can be included in a mechanical system much like a revolute joint. The classical approach, known as zero-clearance approach, assumes that the connecting points of two bodies linked by a revolute joint are coincident. Then the clearance produced in a joint separates these two points. Figure 1 depicts a revolute joint with clearance. The difference in radii between the bearing and journal defines the size of the radial clearance and the radial clearance is defined as follows:

$$c = R_B - R_J \tag{1}$$

where R_B and R_J are the radii of bearing and journal, respectively.

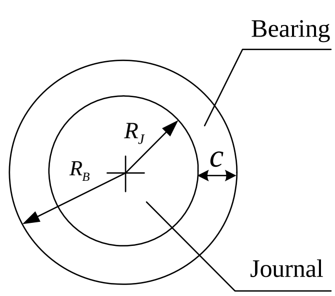


Fig. 1. Schematic of revolute joints with clearance

Although, a revolute joint with clearance does not constrain any degree of freedom from the mechanical system like the ideal joint, it imposes some kinematic restrictions, limiting the journal to move within the bearing. Thus, when clearance is present in a revolute joint, the two kinematic constraints are removed and two degrees of freedom are introduced instead. The dynamics of the joint are then controlled by forces working on the journal and bearing. Thus, whilst a perfect revolute joint in a mechanical system imposes kinematic constraints, a revolute clearance joint leads to force constraints. When contact exists between the journal and bearing, a contact force is applied perpendicular to the plane of collision. Therefore, the motion of mechanical system with clearance always includes contact-impact process.

3. CONTACT FORCE MODELS

The contact force model during the contact process of revolute joint with clearance is one of the important contents of mechanical system. Firstly, this section presents a simple review of the contact force model utilized in this work. In the following, these contact force models are applied to journal and bearing contact in revolute joint with clearance and the contact process between journal and bearing is analyzed by using the different contact force models.

a) Spring-damper model

In the first and simplest model of damper, referred to as spring-damper model, the contact force is represented by a linear spring-damper element. The impact is schematically represented with a linear damper for the dissipation of energy in parallel with a linear spring for the elastic behavior [16]. The contact force model is defined as [17]:

$$F_n = K\delta + b\dot{\delta} \tag{2}$$

where K is spring stiffness coefficient and b is damping coefficient. δ represents elasticity deformation or the relative penetration depth.

In general, the stiffness and damping coefficients have been assumed to be known parameters. The weakness of this contact force model is quantification of the spring stiffness coefficient, which depends on the geometric and material characteristics of the contacting bodies. Besides, the assumption of a linear spring-damper element is a rough approximation for the contact force between two bodies, because the contact force is affected by the shape, surface conditions and mechanical properties of the contacting bodies.

b) Hertz's model

The best-known contact force law is due to the result of pioneering work by Hertz, which is based on the theory of elasticity. The Hertz contact theory is restricted to frictionless surfaces and perfectly elastic solids. This is a non-linear model but limited to impacts with elastic deformation and in its original form does not include damping. With this model, the contact process can be pictured as two rigid bodies interacting via a non-linear spring along the line of impact. The hypotheses used states that the deformation is concentrated in the vicinity of the contact area, elastic wave motion is neglected, and the total mass of each body moves with the velocity of its mass center. The impact force is defined as [15]:

$$F_n = K\delta^n \tag{3}$$

where K and n are constants, depending on material and geometric properties and computed by using elastostatic theory. The exponent n is equal to 1.5 for circular and elliptical contacts. K is the contact stiffness coefficient of the impact body, which is obtained from impact experiment of two spheres and valid for spherical contact only. K is obtained from the following:

$$K = \frac{4}{3\pi(\sigma_i + \sigma_j)}\left[\frac{R_i R_j}{R_i + R_j}\right]^{\frac{1}{2}} \tag{4}$$

$$\sigma_i = \frac{1-v_i^2}{\pi E_i} \qquad \sigma_j = \frac{1-v_j^2}{\pi E_j} \tag{5}$$

where v and E are Poisson ratio and Young modulus, respectively. R_i and R_j are radii of the two spheres. Here, by definition, the radius is negative for concave surfaces, such as for journal element, and positive for convex surfaces, such as for the bearing element.

The advantage of Hertz contact model is that the geometric and material characteristics of the contacting surfaces are considered, which are important for dynamic characteristics analysis of contact. Besides, Hertz contact model is a non-linear relation between the penetration and the contact force. However, the Hertz contact model given by Eq. (3) is limited to contacts with elastic deformations and does not include energy dissipation.

c) Hunt-Crossley contact force model

Although the Hertz law is based on the elasticity theory, some studies have been performed to extend the contact law to include energy dissipation. In fact, the most complicated part of modeling impacts is the process of energy transfer. If an elastic body is subjected to a cyclic load, the energy dissipation due to internal damping causes a hysteresis loop in the force-penetration diagram.

Hunt and Crossley [18] showed that the linear spring-damping model does not represent the physical nature of energy transferred during the impact. Instead, they represent the contact force by the Hertz force-penetration law with a non-linear viscous-elastic element. This approach is valid for direct central and frictionless impacts. The impact force model is defined as:

$$F_n = K\delta^n + b\delta^n\dot{\delta} \tag{6}$$

where b is the damping coefficient related to coefficient of restitution, c_e.

The advantage of Hunt-Crossley model, presented in Eq. (6), is based on Hertz law with a non-linear viscous-elastic element, in which the energy dissipation is included. This contact force model represents the contact process as a non-linear spring-damper model along the direction of collision.

d) Lankarani-Nikravesh contact force model

On the basis of Hunt and Crossley’s work, Lankarani and Nikravesh [19] developed a contact force model with hysteresis damping for impact in multibody systems. The model uses the general trend of the Hertz contact law, in which a hysteresis damping function is incorporated with the intent to represent the energy dissipated during the impact.

Lankarani and Nikravesh [19] suggested separating the normal contact force into elastic and dissipative components. A common expression of Hertz contact force is adopted in Lankarani-Nikravesh model, which considers the effect of damping and describes the energy loss in the contact process. The expression of Lankarani-Nikravesh model is shown in Eq. (7):

$$F_n = K\delta^n + D\dot{\delta} \tag{7}$$

where elastic deformation force is represented by the first item of the right side of Eq. (7) and the energy loss is represented by the second item. δ is the deformation, $\dot{\delta}$ is the relative deformation velocity. K is the contact stiffness coefficient of the impact body, which is obtained from Eq. (4) and (5). Coefficient, D, in Eq. (7) is damping coefficient and $\dot{\delta}$ is relative impact velocity in impact process. The expression of D is shown in Eq. (8):

$$D = \frac{3K(1 - c_e^{\ 2})\delta^n}{4\dot{\delta}^{(-)}} \tag{8}$$

where c_e is coefficient of restitution and $\dot{\delta}^{(-)}$ is initial relative velocity of the impact point.

Due to the coefficient of restitution, c_e, is closed to unity in Eq. (8), the expression obtained is only with higher coefficient of restitution, and the calculation error is higher for the lower coefficient of restitution [9]. Therefore, the contact force model given by Eq. (7) is valid for the cases in which the dissipated energy during the contact is relatively small when compared to the maximum absorbed elastic energy.

e) Hybrid contact force model

The contact force models given by Eqs. (3), (6) and (7) adopted Hertz theory to deal with the contact problem and are only valid for colliding bodies with ellipsoidal contact areas. Hertz theory is available only in solving the contact problem that the geometric shape of contact bodies is non-conformal [3, 15]. However, clearance in actual revolute joint is very small and the contact process of journal and bearing does not always satisfy the non-conformal contact condition. The achieved results are not precise for bearing and journal contact in small clearance.

Bai and Zhao [4] present a new contact force model of revolute clearance joint in planar mechanical system, which is a hybrid model of the Lankarani-Nikravesh model and the improved Winkler elastic foundation model. The contact force model is expressed as:

$$F_n = K_n \delta^n + D_{\text{mod}} \dot{\delta} \tag{9}$$

where the elastic deformation force is represented by the first item of the right side of Eq. (9) and the energy loss is represented by the second item. δ is the deformation, $\dot{\delta}$ is the relative deformation velocity. K_n is the nonlinear stiffness coefficient of the impact body and D_{mod} is the modified damping coefficient.

The nonlinear stiffness coefficient, K_n, is obtained from the following:

$$K_n = \frac{1}{8}\pi E^* \sqrt{\frac{2\delta(3(R_B - R_J) + 2\delta)^2}{(R_B - R_J + \delta)^3}} \tag{10}$$

where R_B and R_J are radii of the bearing and journal, E^* is compound elastic modulus and the expression is represented as:

$$\frac{1}{E^*} = \frac{1 - v_i^2}{E_i} + \frac{1 - v_j^2}{E_j} \tag{11}$$

where v and E are Poisson ratio and Young modulus, respectively. The nonlinear stiffness coefficient, K_n, is related to the material property, geometry property, clearance size and deformation of contact bodies varies with δ, and is not constant during the contact process.

Coefficient D_{mod} in Eq. (9) is modified damping coefficient, which is expressed as Eq. (12):

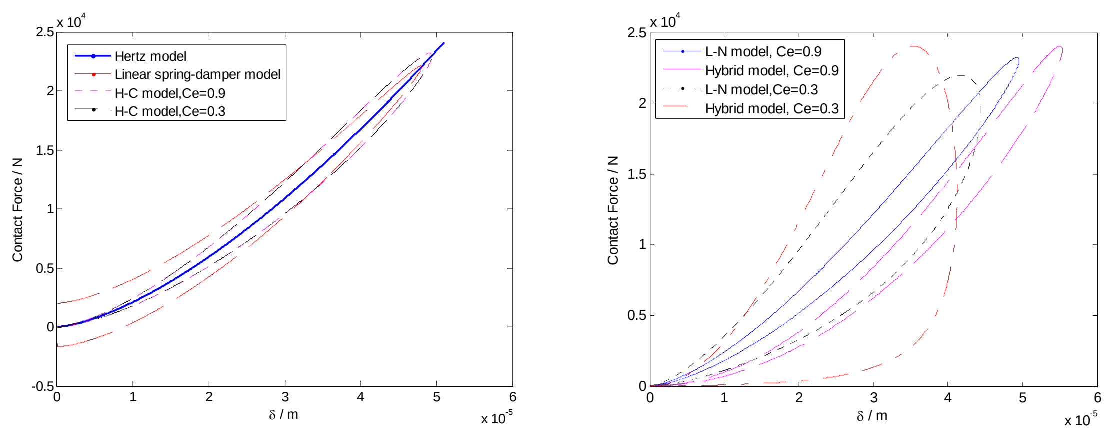
$$D_{\text{mod}} = \frac{3K_n(1 - c_e^{\ 2})e^{2(1-c_e)}\delta^n}{4\dot{\delta}^{(-)}} \tag{12}$$

where c_e is coefficient of restitution and $\dot{\delta}^{(-)}$ is initial relative velocity of the impact point.

f) Contact force model analysis

It must be noted that there are other contact force models that can be considered for contact modeling in revolute clearance joints and this work presents the typical contact force models. Further, the contact responses of journal and bearing in revolute clearance joint are analyzed by using the different contact force models. These contact force models are applied to journal and bearing inter-contact in revolute joint with clearance. The radii of journal and bearing are 1cm and 0.95cm, respectively. The elasticity modulus is $E = 207GPa$ and Poisson ratio is $\upsilon = 0.3$. The mass of journal is 1kg, the initial velocity of journal is 1m/s and the bearing is fixed boundary.

Figure 2 presents the force-deformation curves with different contact force models. From Fig. 2 it can be found that the contact processes are different with the different contact force models. For the linear spring-damper model, given by Eq. (2), the contact force is discontinuous at the beginning of contact because of the damping term. Besides, the damping force does not tally with the actual contact process. In a more realistic model, both elastic and damping forces should initially be at zero and increase over time. Their relative velocity tends to be negative. As a result, a negative force holding the objects together is present. So the linear spring-damping model does not represent the physical nature of energy transferred during the impact. It is apparent that the Hertz's model, given by Eq. (3), is a pure elastic contact model, that is, the contact energy stored during the loading phase is exactly the same restored during the unloading phase, as this model does not take into account the energy dissipation during the process of impact. The great advantage of the Hertz's model relative to spring-damper model is its non-linearity. Since Hertz model does not account for energy dissipation, its equivalent coefficient of restitution is one.



(a) (b)

Fig. 2. Force-deformation curves for different contact force models ((a) Hertz model, linear spring-damper model and Hunt-crossley model; (b) Lankarani-Nikravesh model and Hybrid model)

Hunt-Crossley contact force model, given by Eq. (6), represents the contact force by the Hertz force-penetration law with a non-linear viscous-elastic element. Therefore, Hunt-Crossley contact force model is non-linear and considers energy dissipation during the process of impact. As shown in Fig. 2, it represents the hysteresis damping characteristic and reflects the energy dissipation during contact process. For the Lankarani-Nikravesh contact model given by Eq. (7), when the coefficient of restitution is equal to unit, which corresponds to the pure Hertz contact law, there is no energy dissipation in the contact process. This fact is evident in the force-deformation diagram of Fig. 2, which does not present a hysteresis loop. When the coefficient of restitution is higher, i.e. the coefficient of restitution is close to unity, the energy dissipation of hybrid model, given by Eq. (9), and Lankarani-Nikravesh contact model given by Eq. (7), both reflect the energy dissipation during the contact process of revolute joint with clearance accurately. However, when the coefficient of restitution is lower, the contact process is very different, and the energy dissipation of the hybrid contact model better reflects the energy dissipation during the contact process, reflecting large energy dissipation. Therefore the results are improved. It is found that the Hunt-Crossley contact force model and the Lankarani-Nikravesh contact force model can obtain better precision for higher coefficient of restitution. However, the hybrid contact force model has better calculation precision for both lower and higher coefficient of restitution and has a greater applicable scope.

Further, an output coefficient of restitution for different contact force models utilized can be evaluated as:

$$c_{eout} = -\frac{\dot{\delta}^{(+)}}{\dot{\delta}^{(-)}} \tag{13}$$

where c_{eout} represents the actual output coefficient of restitution for each of the different contact force models utilized. $\dot{\delta}^{(-)}$ is initial relative approach normal velocity and $\dot{\delta}^{(+)}$ is the actual departing normal velocity of the impact point. This actual measure of the coefficient of restitution is different from the initial coefficient of restitution c_e used in the contact force expression. The actual output coefficients of restitution are 1, 0.9132, 0.7252 and 0.6842, when the Lankarani-Nikravesh contact force model is used with the initial coefficients of restitution c_e which are 1, 0.9, 0.5 and 0.3, respectively. It can be found that the Lankarani-Nikravesh contact force model results in a larger amount of restitution. Therefore, the Lankarani-Nikravesh contact force model is closer to the actual value for more elastic impact. The difference between the two is due to the assumption that the dissipated energy in impact is small compared to the maximum stored elastic energy [19]. However, the actual output coefficients of restitution are 1, 0.8959, 0.4826 and 0.3267, when the hybrid contact force model is used with the initial coefficients of restitution c_e are 1, 0.9, 0.5 and 0.3, respectively. Comparing the actual output coefficients of restitution calculated by Lankarani-Nikravesh contact force model and the hybrid contact force model, it also can be found that the hybrid contact force model has better calculation precision for the amount of restitution for both lower and higher coefficient of restitution.

4. FRICTION FORCE MODEL

It is known that the Coulomb law of sliding friction can represent the most fundamental and simplest model of friction between dry contacting surfaces. The tangential contact characteristic of clearance joint is represented using tangential friction force model. Thus, in this paper, the friction effects in joints are considered as dry friction and a modified Coulomb friction model is used to represent the friction response between the journal and bearing. Friction coefficient, which is not a constant, is introduced in the modified Coulomb friction model. Friction coefficient is a function of tangential sliding velocity, which can represent the friction response in impact and contact process as well as the viscous and micro-slip phenomenon in relative low-velocity case more accurately. Moreover, the modified Coulomb friction model can avoid the case of abrupt change of friction in numerical calculation as the change of velocity direction.

The expression tangential friction force is shown as Eq. (14):

$$F_t = -\mu(v_t) F_n \frac{\boldsymbol{v}_t}{|\boldsymbol{v}_t|} \tag{14}$$

where friction coefficient $\mu(\boldsymbol{v}_t)$ is a function of tangential sliding velocity and which can be expressed as:

$$\mu(v_t)=\begin{cases} -\mu_d sign(v_t) & for \quad |v_t|>v_d \\ -\left\{\mu_d+(\mu_s-\mu_d)\left(\dfrac{|v_t|-v_s}{v_d-v_s}\right)^2\left[3-2\left(\dfrac{|v_t|-v_s}{v_d-v_s}\right)\right]\right\}sign(v_t) & for \quad v_s\le|v_t|\le v_d \\ \mu_s-2\mu_s\left(\dfrac{v_t+v_s}{2v_s}\right)^2\left(3-\dfrac{v_t+v_s}{v_s}\right) & for \quad |v_t|<v_s \end{cases} \tag{15}$$

where v_t is relative sliding velocity in the collision point of journal and bearings, which is the velocity component in tangential direction. μ_d is dynamic friction coefficient. μ_s is static friction coefficient. v_s is critical velocity of static friction. v_d is critical velocity of the maximum dynamic friction.

5. DEMONSTRATIVE APPLICATION EXAMPLE

a) Slider-Crank mechanism with revolute clearance joint

In this section, the dynamics characteristics of multibody mechanical systems with revolute clearance joint are investigated using different contact force models. The planar slider-crank mechanism [20] with revolute clearance joint is used as numerical example to demonstrate and validate the contact models presented in this work. The dynamic response obtained with experiment is compared with that of the numerical models to validate the effectiveness of the contact models presented for mechanical system.

Figure 3 depicts the kinematic configuration of the slider-crank mechanism, which consists of four bodies, including ground, two ideal revolute joints, and one ideal translational joint. A revolute clearance joint exists between the connecting rod and slider. The clearance size is 0.25mm. In order to keep the analysis simple and to illustrate the dynamic clearance joint behaviour, all the bodies are considered to be rigid. The length and inertia properties of the slider-crank mechanism components are listed in Table 1 and the parameters used in the dynamic simulations are given in Table 2. In the dynamic simulation the crank is the driving body and rotates with a constant angular velocity equal to 200r/min. The initial configuration corresponds to crank and connecting rod collinear and the position and velocity journal centers are taken to be zero.

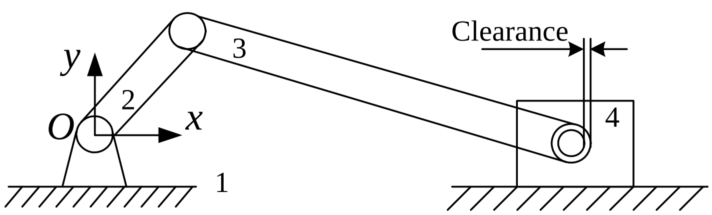


Fig. 3. Slider-crank mechanism with a clearance joint

Table 1. Geometric and inertia properties of the crank-slider mechanism

Body	Length(m)	Mass(kg)	Moment of inertia(kgm^2)
Crank	0.05	17.900	0.460327
Connection rod	0.3	1.130	0.015300
Sliding block	-	1.013	0.000772

Table 2. Simulation parameters for the experimental slider-crank mechanism

Restitution coefficient	0.46
Dynamic friction coefficient	0.01
Young’s modulus	207GPa
Poisson’s ratio	0.3

b) Results and discussion

The effects of using different contact force models on the dynamic responses of the slider-crank mechanism are discussed in this section. The simulation results are compared with the experimental results [20]. Figure 4 presents the slider acceleration using different contact force models.

Figure 4a depicts the slider acceleration evolution when the linear spring-damper model is utilized, where K is 1.0549×10^{11} N/m and b is 1×10^{3} Ns/m. From Fig. 4a, it is clear that the slider acceleration with clearance is obviously shaking with very high peaks when compared with the experimental data. The slider acceleration obtained from computational simulation is significantly different from those obtained from experiment [20]. Figure 4b depicts the slider acceleration evolution when the Hertz model is utilized. The peaks of slider acceleration from Hertz contact model are lower when compared with those of the linear spring-damper model. However, the slider acceleration is also different from the experimental data. It can also be found that considering the significant improvements in terms of the reduction of the acceleration peaks it is reasonable that the non-linear relation between the contact force and penetration will improve the performance of the model's response in terms of the correlation between the experimental and computational results. Figures 4c, 4d and 4e depict the slider acceleration evolution when the Hunt-Crossley model, Lankarani-Nikravesh model and the hybrid model are utilized, respectively. In a similar manner, the outcomes from the three contact force models represent the same evolution and the slider acceleration peaks are lower when compared with the Hertz model. It indicates that taking into account the significant improvements in terms of the reduction of the acceleration peaks. Although the dissipative item for each contact force model is different, all three dissipative contact force models produce nearly similar results on the dynamics of the slider crank mechanism. It is reasonable that the inclusion of the damping terms to the contact approach will improve the performance of the model's response. It should be noted that when these three contact force models, that is Hunt-Crossley model given by Eq. (6), Lankarani-Nikravesh model given by Eq. (7) and the hybrid model given by Eq. (9) are used respectively, the obtained results match reasonably well with the experimental data, in particular for the hybrid model utilizing modified damping term given by Eq. (12). This is not surprising since these three approaches include some damping in terms of the restitution coefficient. Figures 4c and 4d clearly show that the Hunt-Crossley model and Lankarani-Nikravesh model provides some significant improvements over the pure elastic force laws. This might be taken as an indicator that damping does indeed play a crucial role in these types of contact event. However, the predicted peak values are a little higher than the experimental dates. This is because the Hunt-Crossley model and Lankarani-Nikravesh model adopt Hertz theory and are only valid for colliding bodies with ellipsoidal contact areas, as well as the damping term included these contact force models. This drawback is overcome by observing the plots of Fig. 4e, in which the hybrid model and experimental data match quite well. Therefore, the hybrid contact force model together with the modified damping term will predict a better response of mechanical system with revolute clearance joint.

The same phenomena can be observed from the contact force curve in revolute joint based on the different contact force models presented by Fig. 5. As shown in Fig. 5, the existence of clearance leads to the impact force in the joint increase and the impact force is high-frequency vibration. From Fig. 5a, it can be seen that the contact force evolution using the spring-damper model is obviously shaking with very high peaks. Figure 5b depicts the contact force evolution when the Hertz model is utilized. The peaks of contact force from Hertz contact model are lower when compared with those of the linear spring-damper model. Figures 5c, 5d and 5e depict the contact force evolution when the Hunt-Crossley model, Lankarani-Nikravesh model and the hybrid model are utilized, respectively. The force peaks based on these models are lower when compared with the Hertz model. The same phenomena can be observed in the curve of slider acceleration represented by Fig. 4. It should be noted that these three approaches include some damping in terms of the restitution coefficient which indicates that the inclusion of the damping terms to the contact approach will lead to better results. Figures 5c, 5d 5e clearly show that the Hunt-Crossley model, Lankarani-Nikravesh model and the hybrid model provide some significant improvements over the pure elastic force laws. Besides, the same phenomena can also be observed in the curve of crank moment, which is required to maintain the crank angular velocity constant and is presented by Fig. 6. It clearly shows some significant improvements in terms of the reduction of the crank moment peaks. It is reasonable that the inclusion of the damping terms to the contact approach will improve the performance of the model's response. Therefore, it is noted that damping does indeed play a crucial role in these types of contact event and it describes the contact process of revolute joint with clearance reasonably.

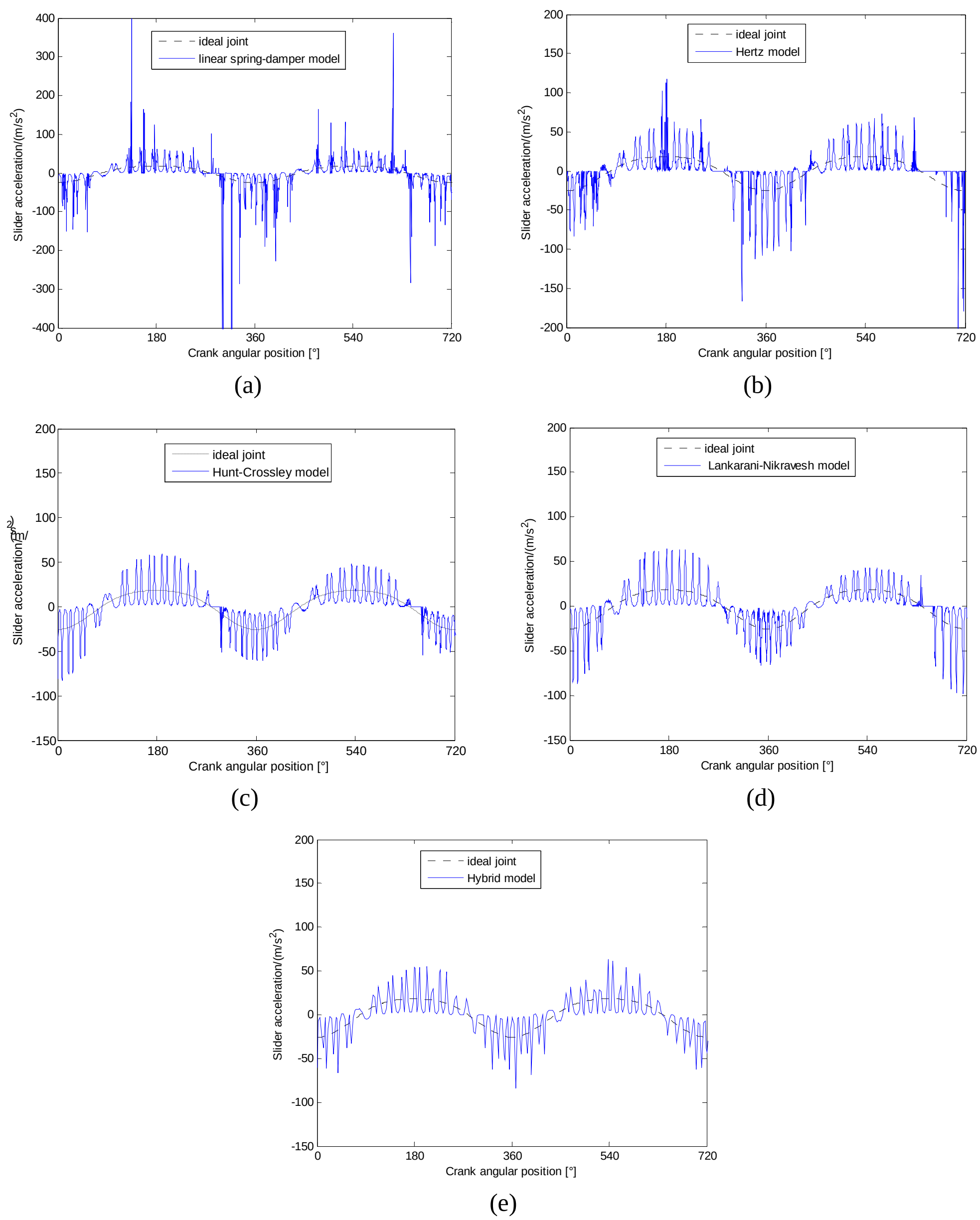


Fig. 4. Slider acceleration using different contact force models ((a) Linear spring-damper model; (b) Hertz model; (c) Hunt-Crossley model; (d) Lankarani-Nikravesh model; (e) Hybrid model)

The fact that the contact force model of the clearance joint has an important effect on the dynamics characteristics supports the idea that the selection of appropriate contact force model of clearance joints must be considered in the analysis and design of the real mechanical system.

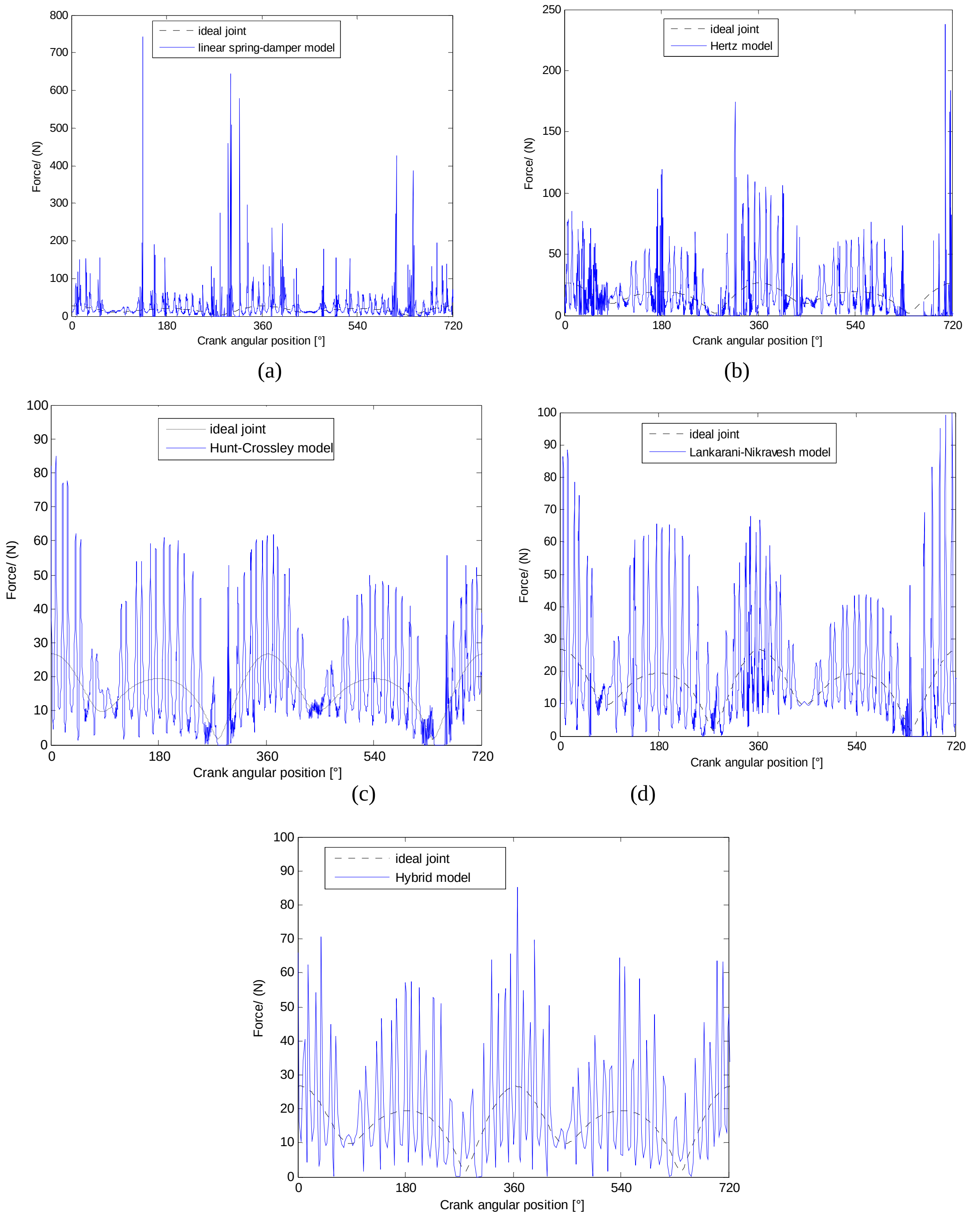


Fig. 5. Contact force in clearance joint using different contact force models ((a) Linear spring-damper model; (b) Hertz model; (c) Hunt-Crossley model; (d) Lankarani-Nikravesh model; (e) Hybrid model)

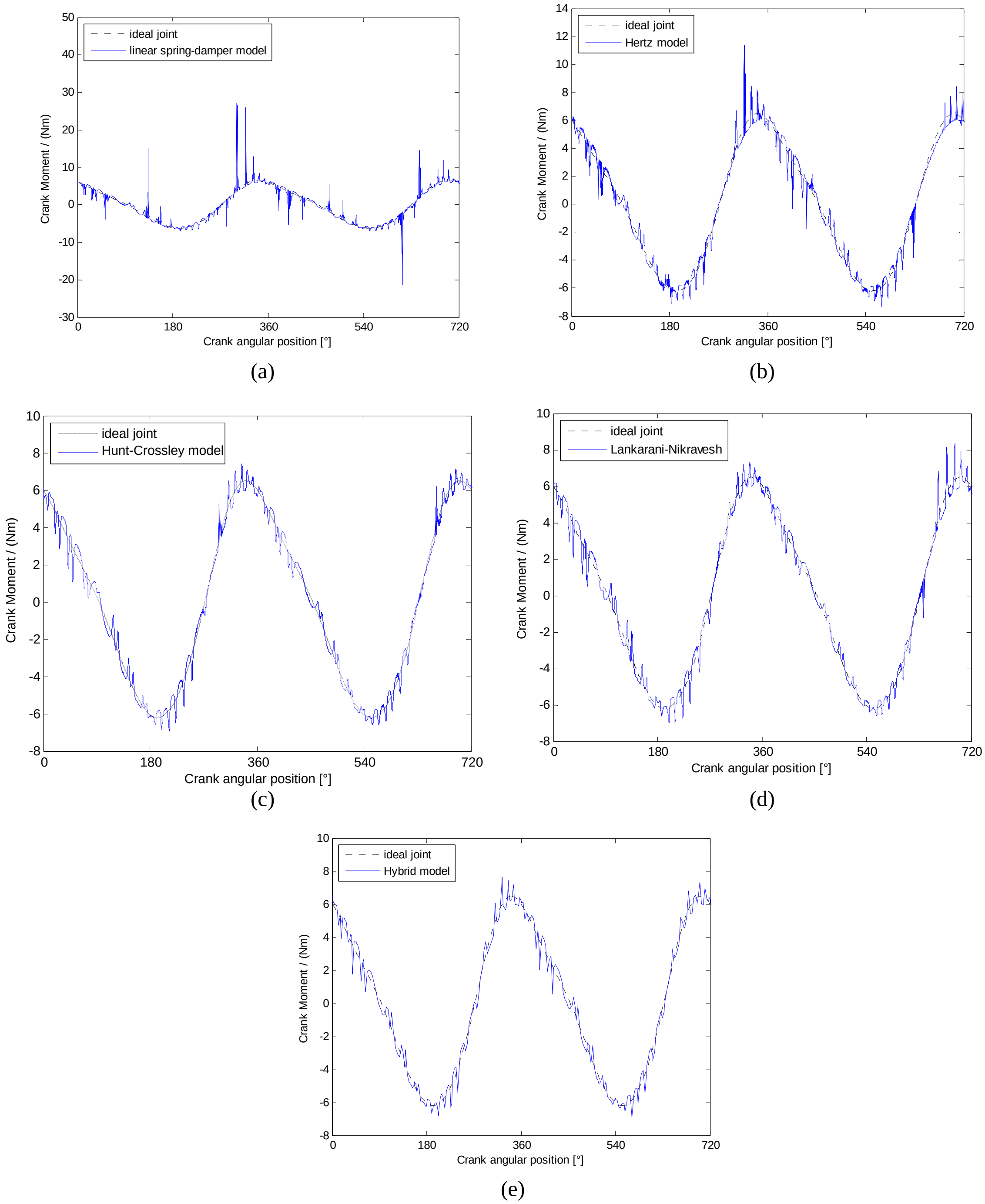


Fig. 6. Crank moment using different contact force models ((a) Linear spring-damper model; (b) Hertz model; (c) Hunt-Crossley model; (d) Lankarani-Nikravesh model; (e) Hybrid model)

6. CONCLUSION

The effects of contact force models on dynamics responses of multibody mechanical system with revolute clearance joint are investigated using a computational method. The intra-joint contact forces that are generated at these clearance joints are computed by considering several different elastic and dissipative approaches. A simple review of the constitutive laws utilized in this work is presented and analyzed.

Finally, a well-known slider-crank mechanism with a revolute clearance joint is utilized to perform the investigation.

The existence of clearance in revolute joints leads to contact and impact force and causes the dynamic characteristics of the system to change. The acceleration of mechanical system with clearance is obviously shaking and the amplitude increases from the mechanism without clearance. The same conclusion can also be drawn from curve of input crank moment, which is required to maintain the crank angular velocity constant. It indicates that the effects of clearance on the dynamic characteristics of mechanism cannot be ignored. Based on the general results obtained from computational analysis and compared with experimental data, the contact process of journal and bearing in revolute clearance joint and the dynamics responses of the mechanical system are different when using different contact force models. It is seen that the effects of contact force model on the dynamics responses of mechanical system cannot be ignored. Although the dissipative item for each contact force model is different, the inclusion of the damping terms to the contact approach will improve the performance of the response. Therefore, damping in contact approach does indeed play a crucial role in dynamics of mechanical system. In addition, the hybrid contact force model with a modified damping term provides results more reasonably. Overall, the contact force model of the clearance joint has an important effect on the dynamics responses of mechanical system, which supports the idea that the selection of appropriate contact force model of clearance joints plays a significant role in the dynamics analysis and design of multibody mechanical system with revolute clearance joint.

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