IJST, Transactions of Mechanical Engineering, Vol. 35, No. M1, pp 1-13
Printed in The Islamic Republic of Iran, 2011

A NON MODEL-BASED DAMAGE DETECTION TECHNIQUE USING DYNAMICALLY MEASURED FLEXIBILITY MATRIX*

M. SALEHI[1], S. ZIAEI RAD[1], ** M. GHAYOUR[1] AND M.A. VAZIRY[2]

[1]Dept. of Mechanical Engineering, Isfahan University of Technology, Isfahan, I. R. of Iran
[2]Dept. of Aerospace Engineering, Amirkabir University of Technology, Tehran, I. R. of Iran
Email: szrad@cc.iut.ac.ir

Abstract– Although a numerous methods exist for detecting damage in a structure using measured modal parameters, many of them require a correlated finite element model, or at least, modal data of the structure for the intact state as the baseline. For beam-like structures, curvature techniques, e.g., mode shape curvature and flexibility curvature have been applied to localize damage. An approximate flexibility matrix can be derived using a small number of vibration modes whose columns are referred to as flexibility shapes. In this paper, a damage localization method based on changes in flexibility shapes as well as its curvature is developed. Differential Quadrature Method (DQM) has been implemented to obtain the curvatures of flexibility shapes. It was shown that the method works well using the damaged model alone. To reduce the potential error due to differentiation of experimental data, a moving curve fit method has been implemented on flexibility shapes. The capability of the proposed method is demonstrated through numerical and experimental case studies.

Keywords– Modal analysis, modal testing, damage detection

1. INTRODUCTION

Damage generally produces changes in the physical properties of a structure (i.e., stiffness, mass, and damping), and these changes are accompanied by changes in the modal characteristics of the structure (i.e., natural frequencies, mode shapes, and modal damping). This phenomenon has been widely noted and used by structural engineers for structural damage detection. Sohn et al. [1] provided an excellent review on research advances in this area over the last 30 years, and summarized this kind of technology as vibration-based damage identification methods.

According to the process, to treat the measured modal parameters, the vibration-based damage identification methods can be classified as model based and non model-based. The model-based methods identify damage by correlating an analytical model, which is usually based on the finite element theory, with the modal test data of the damaged structure [2, 3]. Comparisons of the updated model to the original one provide an indication of damage and further information on the damage location and/or its extent. However, the construction of the finite element model usually gives rise to model errors from simplified assumptions. To detect the damage other than the artificial errors from the model construction, a good quality finite element model that could accurately predict the behavior of the intact structure is required, but is often difficult to achieve.

Non model-based damage detection methods, also called damage index methods, are relatively straightforward. The changes in the modal parameters between the intact and damaged states of the

---

*Received by the editors June 21, 2009; Accepted March 1, 2011.
**Corresponding author

structure are directly used or correlated with other relevant information, to develop the damage indicators for localizing damage in the structure. Early works of damage index methodology make use of the natural frequency and mode shape information. Shifts in the natural frequencies [4], changes in the modal assurance criterion (MAC) across sub-structures [5], changes in the co-ordinate modal assurance criterion (COMAC) [6], and changes in the multiple damage location assurance criterion (MDLAC) [7] between the intact and damaged structures are formulated as damage indicators. Pandey et al. [8] further demonstrated that changes in mode shape curvature could be a good indicator of damage for beam-like structures.

During the last decade, some researchers found that the modal flexibility can be a more sensitive parameter than natural frequencies or mode shapes alone for structural damage detection. Raghavendrachar and Aktan [9] examined the application of modal flexibility for a three span concrete bridge. In their comparison with natural frequency and mode shapes, the modal flexibility is reported to be more sensitive and reliable for local damages. Zhao and Dewolf [10] presented a theoretical sensitivity study comparing the use of natural frequencies, mode shapes, and modal flexibility for damage detection. The results demonstrate that modal flexibility is more likely to indicate damage than the other two. Pandey and Biswas [11] proposed a damage localization method based on directly examining the changes in the measured modal flexibility of a beam structure.

Zhang and Aktan [12] comparatively studied the modal flexibility and its derivative, called uniform load surface (ULS), for their truncation effect and sensitivity to experimental errors. They suggested that the ULS has much less truncation effect and is least sensitive to experimental errors.

In this paper, a structural damage localization approach is proposed based on information about damaged structure. The technique does not require an analytical model or intact state information. Some case studies were selected to demonstrate the capability of the method for both simulated and experimental data.

2. THEORY

The basic premise of vibration based damage detection methods is that the damage alters the mass, stiffness and damping properties of a structure, which in turn affects structural modal parameters. Stiffness change due to damage is a promising factor in damage localization, because any changes in elemental stiffness throughout the stiffness matrix can lead to structural damage location. Unfortunately, derivation of a fairly accurate stiffness matrix based on a few lower frequency measured modal data is very difficult to achieve. However its inverse, i.e., flexibility matrix, can be approximated using the first few measured modal data due to the inverse relationship to the square of modal frequencies [13].

With the mass normalized modes ϕ_i, the stiffness matrix K and the flexibility matrix F can be expressed by mode expansion using n modes, each having m degrees of freedom:

$$K = \Phi \Omega \Phi^T = \sum_{i=1}^{n} \omega_i^2 \phi_i \phi_i^T \tag{1}$$

$$F = \Phi \Omega^{-1} \Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T \tag{2}$$

Where $\Phi_{m \times n} = [\phi_1 \; \phi_2 \; ... \; \phi_n]$ and $\Omega = diag\left(\omega_1^2, ..., \omega_n^2\right)$ are the mode shape and eigenvalue matrix, respectively. As evident from Eqs. (1) and (2), the modal contribution to the stiffness matrix increases with higher modes. Adversely, the modal contribution to the flexibility matrix decreases as the frequency increases, so it converges using a few lower modes.

Each column of the flexibility matrix represents the displacement pattern of the structure associated with a unit force applied at the associated DOF. In the damaged vicinity of a uniform beam, an abnormality occurs in the aforementioned displacement shape due to sudden stiffness change. This abnormality can also be seen in mode shapes, however, flexibility shapes seem to exhibit the damage better, because they make use of frequency information as well as mode shapes. Besides, available mode shapes are usually few and if the damage location coincides with the nodal points, the corresponding mode shape cannot be damage indicative. But in the flexibility based method, the number of shapes used in the damage localization is equal to the number of measurement grid points.

Two methods will be introduced to discover the sharp points of structural damage position in flexibility shapes, i.e., shape curvature and gapped smoothing method.

Differential Quadrature Method (DQM) has been applied to calculate the shape curvatures. The DQM is an efficient and accurate numerical method and is frequently used for solving non-linear partial differential equations. The main feature of DQM is that the partial derivatives of a function can be numerically evaluated by multiplication of a weighting function and nodal values. If the function $f(x_i)$ is given, its derivatives, with respect to a spatial variable at any discrete point can be mathematically expressed as [14]

$$f^{(n)}(x_i)=\sum_{r=1}^{m} C_{ir}^{(n)} f(x_r) \tag{3}$$

Where m is the number of measurement grid points, $C_{ir}^{(n)}$ are the weighting coefficients associated with the nth order derivatives of $f(x)$ with respect to x at point x_i. The weighting coefficients can be obtained using the following recurrence formulae

$$C_{ir}^{n}=n\left(C_{ii}^{(n-1)}C_{ir}^{(1)}-\frac{C_{ir}^{(n-1)}}{x_i-x_r}\right) \quad i,r=1,2,...,m\,, r\neq i\,, n=2,3,...,m-1 \tag{4}$$

$$C_{ii}^{(n)}=-\sum_{\substack{r=1\\ r\neq i}}^{m} C_{ir}^{(n)} \quad i=1,2,...,m \;\;, n=2,3,...,m-1 \tag{5}$$

$$C_{ir}^{(1)}=\frac{M^{(1)}(x_i)}{(x_i-x_r)M^{(1)}(x_r)} \quad i,r=1,2,...,m\,, r\neq i \tag{6}$$

Where

$$M^{(1)}(x_i)=\prod_{\substack{r=1\\ r\neq i}}^{m}(x_i-x_r)$$

Gapped Smoothing Method (GSM) is another method for locating abnormalities in flexibility shapes.

At the damaged node, the flexibility shape exhibits a local non-smoothness or curvature discontinuity. The GSM is implemented to quantify this behavior at each node [15]. Figure (1) depicts a schematic of GSM. Line fitting through points before and after a specific node and subtracting line fitted prediction and the original value comes to a definition of index δ_i, indicating the relative degree of abnormality of node i. An alternative approach is using cubic polynomial fitting through four data points. Although the latter is of higher accuracy, it suffers from two drawbacks. Firstly, the damage detection at the boundaries of data intervals is fairly poor since the lack of data in these regions forces the use of extrapolation, which is of lower accuracy. Secondly, its potential damage indication interval is wider compared to that of the linear GSM. In this paper, both linear and cubic polynomials are implemented in case studies.

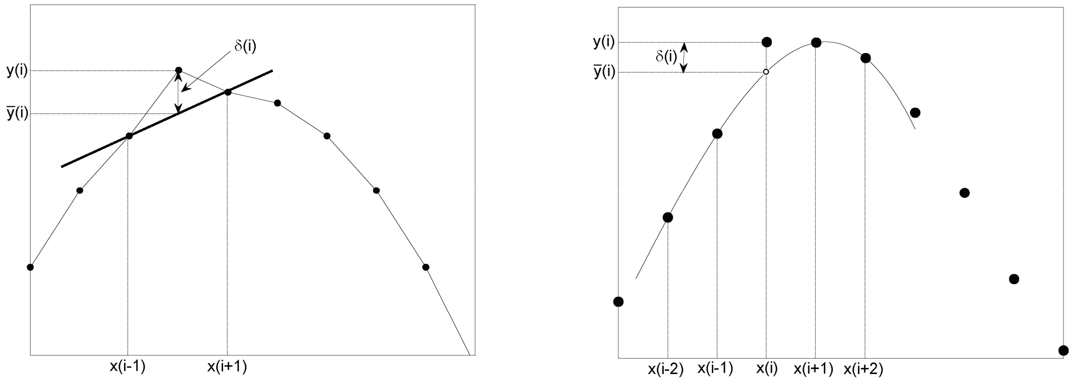

Fig. 1. Non-smoothness evaluation at node i via linear (left) and cubic (right) GSM

3. NUMERICAL CASE STUDY

A 0.4m length aluminum cantilever beam of the cross section 8×40 mm^2 is discretized by 40 two dimensional beam elements, each having six degrees of freedom. Five flexural modes are considered for computation of the flexibility matrix. The numerical model is shown in Fig. 2.

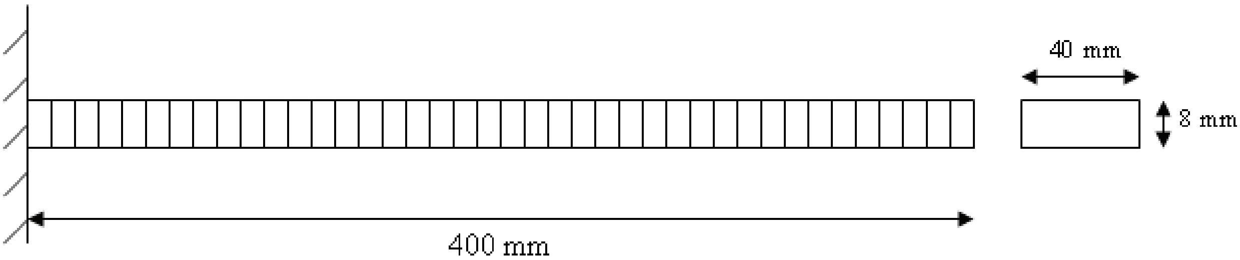

Fig. 2. Simulated cantilever beam

Damage is artificially modeled in four cases as shown in Table (1).

Table 1. Damage cases

Case	Damage
A	4mm thickness reduction in element 6
B	4mm thickness reduction in element 11
C	4mm thickness reduction in element 36
D	2mm thickness reduction in element 11

Figure 3 depicts the first four mode shapes of the structure. There is no considerable difference found between the mode shapes of the damaged and undamaged cases, especially in the lower modes.

Curvature of flexibility shapes was calculated using DQM for the damaged cases. Figure 4 shows the absolute curvatures. It is seen from the figures that the damage region is easily detectable using the flexibility matrix of the damaged structure except for the case C. The results will be more obvious if the curvature difference between the intact and damaged structure is considered.

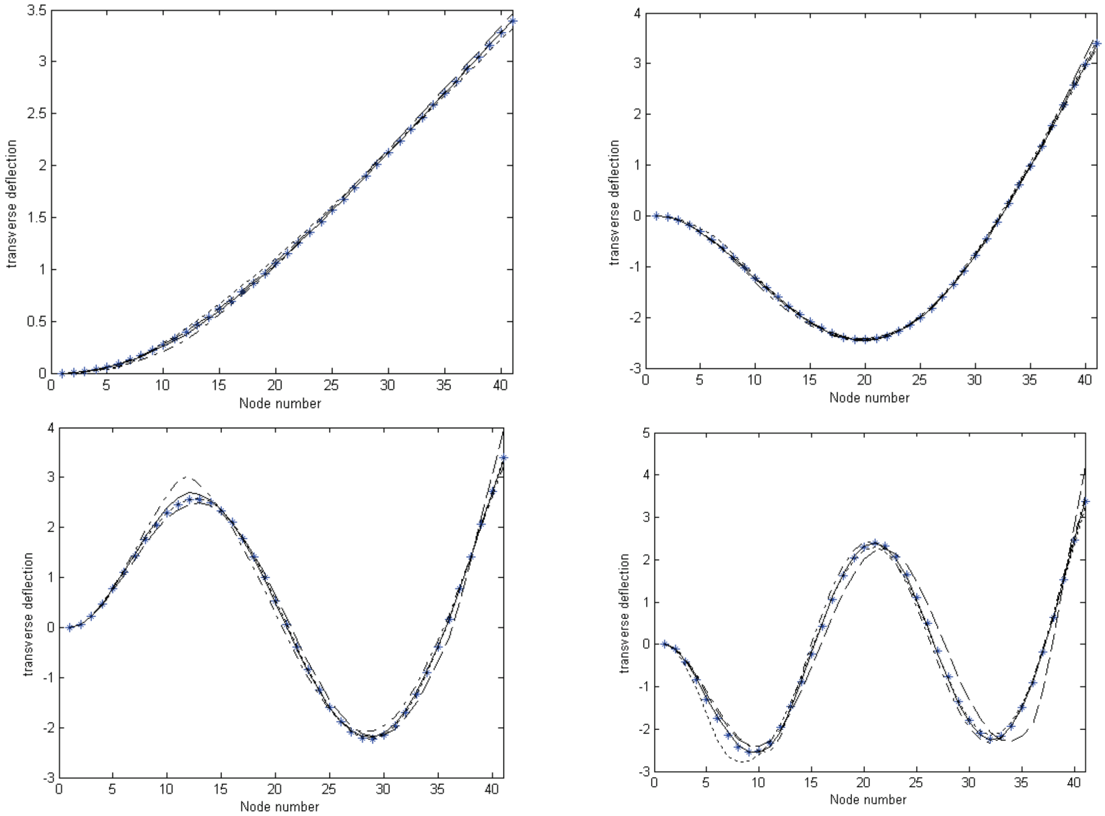

Fig. 3. The first four mode shapes of intact and damaged structures: *undamaged, …damage case A, -.-damage case B, - -damage case C, ─damage case D

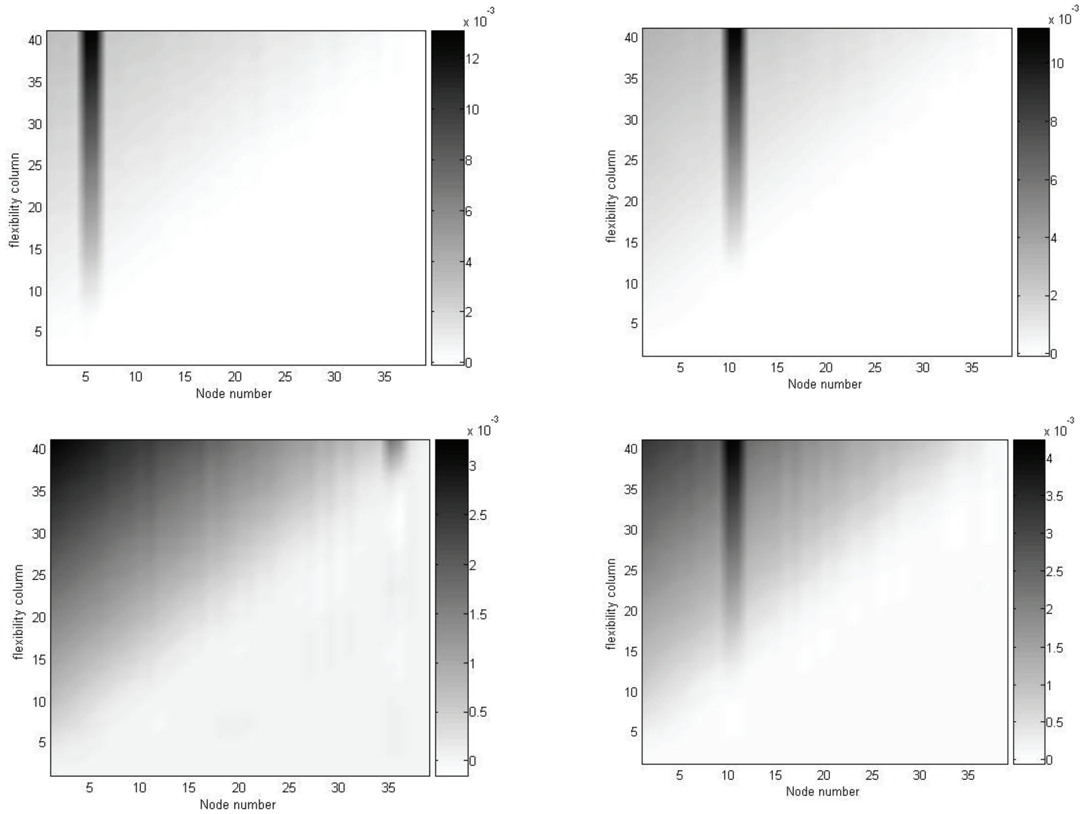

Fig. 4. Curvature of flexibility shapes: case A (top-left), case B (top-right), case C (bottom-left), case D (bottom-right)

The GSM approach was also implemented on damaged state flexibility shapes. Two kinds of linear and cubic polynomials have been considered to catch the irregular points. Figure 5 shows the results of linear GSM for damage cases.

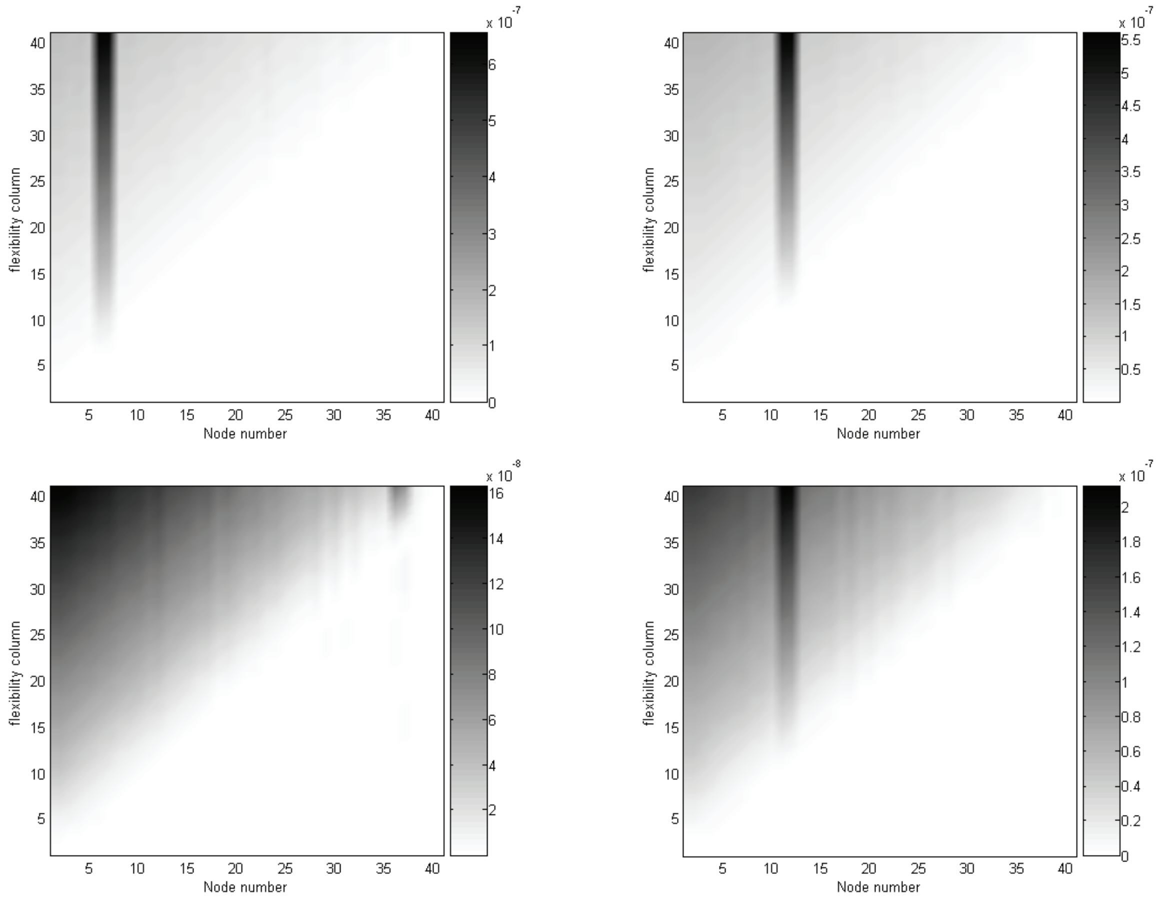

Fig. 5. Damage localization via linear GSM: case A (top-left), case B (top-right), case C (bottom-left), case D (bottom-right)

As can be seen, except for case C, the damaged regions have been well localized using linear GSM. Comparing cases B and D, it can be concluded that in the more severe damage, the method works better.

As mentioned earlier, higher order GSM has the potential to identify the damaged region better in the penalty of wider probable damaged region. Figure 6 shows the results of applying cubic GSM.

To study the sensitivity of damage indication method to damage position, a 4mm thickness reduction was moved along the beam. Curvature of the first four mode shapes was studied instead of flexibility shapes. Due to the linear dependence of flexibility shapes to mode shapes, the sensitivity conclusion about the modes will also be valid for flexibility shapes. Figure 7 depicts the first four mode shapes, their curvature in the intact state and curvature difference between intact and each moving damage case. Curvatures have been scaled for convenient inspection. As is evident, irregularity amplitude in each damaged position for all modes is in straight relation with the absolute curvature in each location. In other words, if the damage has occurred in a region of large amplitude curvature of all modes, the flexibility shape damage indication method works better. That is why the damaged region of case C in the previous example could not be recognized accurately.

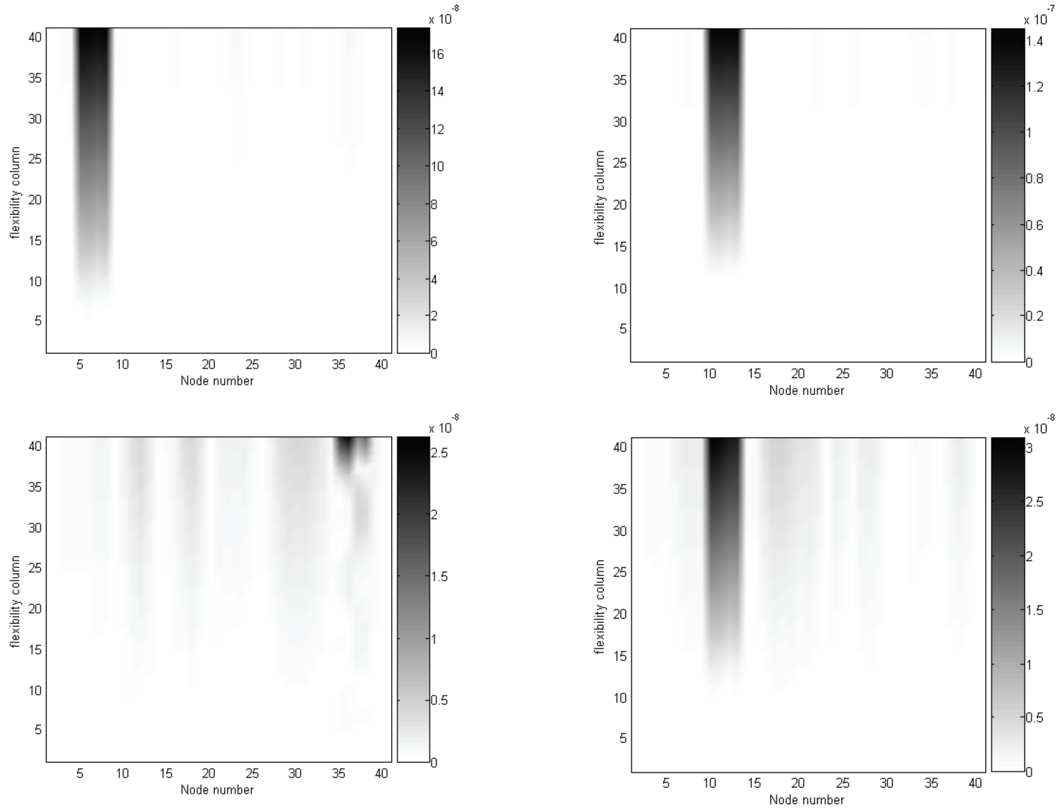

Fig. 6. Damage localization via cubic GSM: case A (top-left), case B (top-right), case C (bottom-left), case D (bottom-right)

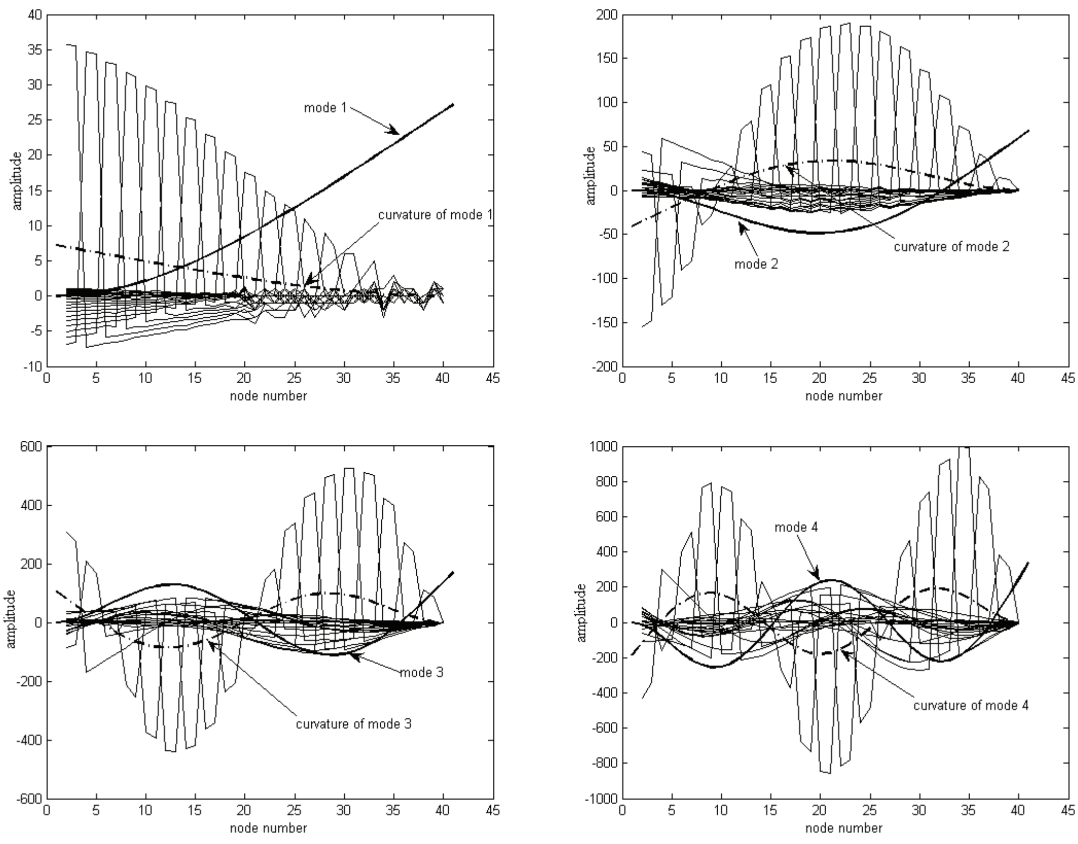

Fig. 7. Curvature difference between intact and damaged states (solid lines) for moving damage over elements, mode shape (dashed line) and mode shape curvature (dotted line): first mode (top-left), second mode (top-right), third mode (bottom-left) and fourth mode (bottom-right)

4. EXPERIMENTAL CASE STUDY

To verify the proposed method from a practical point of view, modal test and analysis were carried out on a rectangular cross section aluminum beam. The test was carried out in free-free condition by suspending the test object by some elastic bungees. The beam is of cross section 50×25 millimeters and 800 millimeters length. Seventeen equally spaced measurement points were marked on the beam. The test was carried out in the frequency range of 0 to 1600 Hz. A B&K 8202 hammer with metal tip was used to excite the structure and acceleration response was measured by a PCB 356B08 accelerometer. The FFT analyzer and modal analysis software used to carry out the test and process the frequency response functions were B&K Portable PULSE 3560D and ICATS software respectively. Figure 8 shows the test setup and instrumentations used for the experimental case study.

Fig. 8. Experimental test setup

Table 2 describes the damage cases considered.

Table 2. Modal test cases

Case	Damage description
A0	No damage
A1	10mm deep slot of 8mm width on node 7
A2	14mm deep slot of 8mm width on node 7
A3	18mm deep slot of 8mm width on node 7

Figure 9 shows a schematic of frequency response function processing to get the modal parameters for case A1. Multi-FRF Global-M method has been used to extract modal parameters.

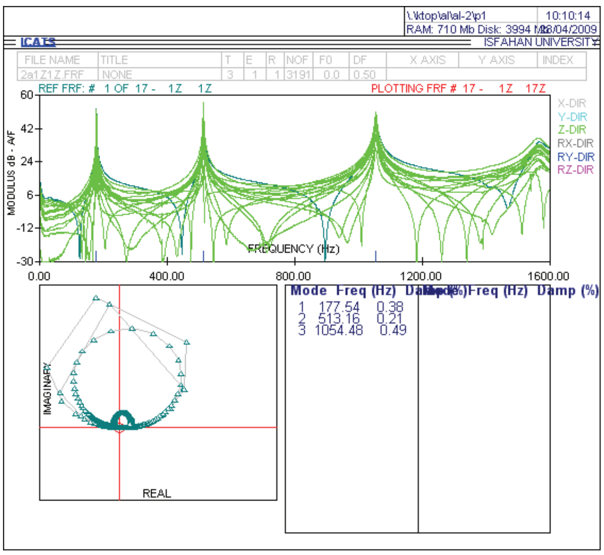

Fig. 9. Modal analysis of measured FRFs

To avoid errors due to material and geometry discrepancies, all tests were made on the same beam. The first three elastic extracted modes from the experimental data are depicted in Figs. 10 to 12.

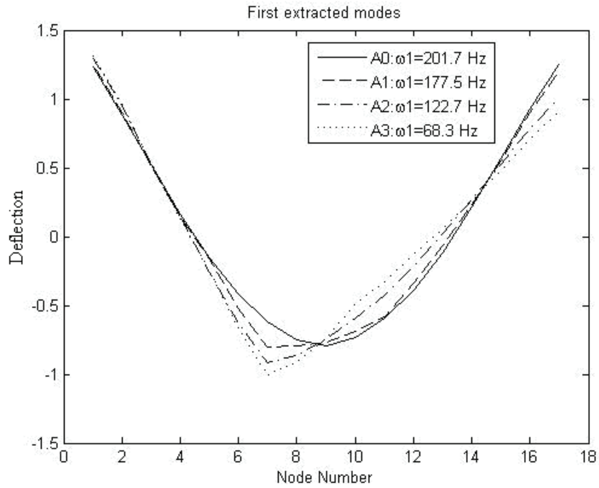

Fig. 10. First modes of experimental cases

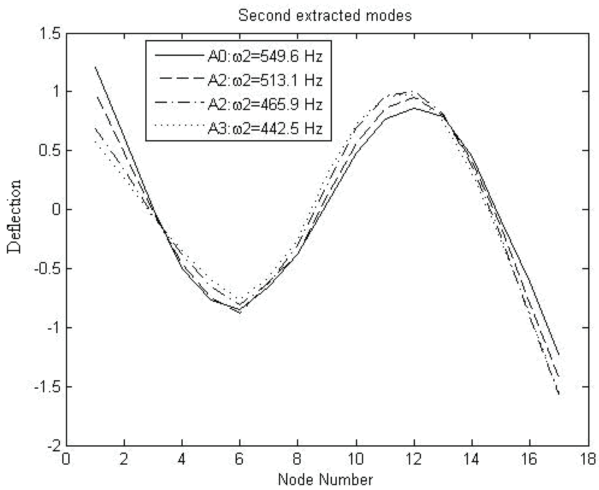

Fig. 11. Second modes of experimental cases

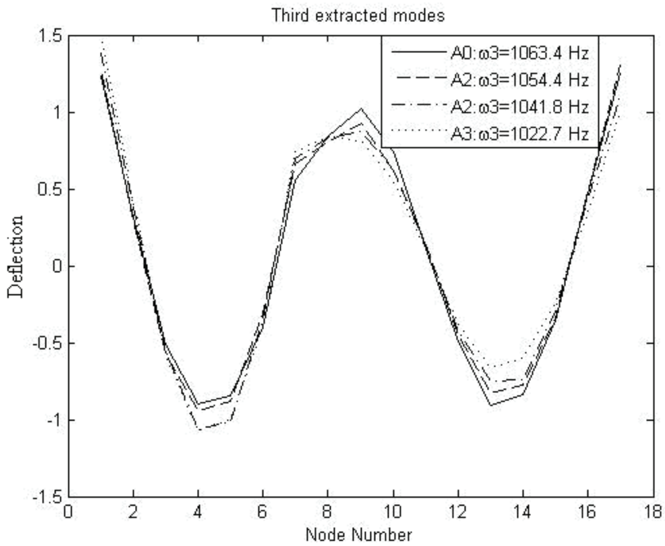

Fig. 12. Third modes of experimental cases

Flexibility matrix for each damage case was calculated using Eq. (2) and then the curvature and GSM approaches were implemented on them. Figures 13 to 15 show the results of the curvature method.

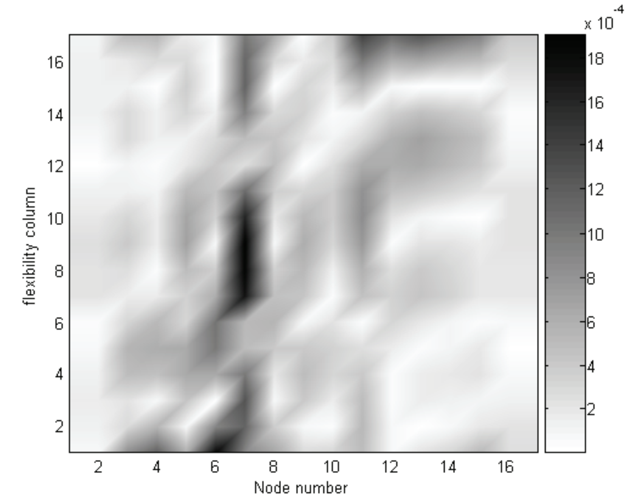

Fig. 13. The curvature of flexibility shapes: case A1

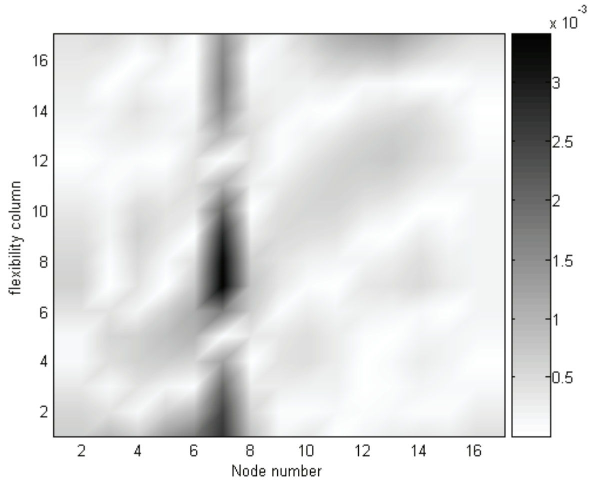

Fig. 14. The curvature of flexibility shapes: case A2

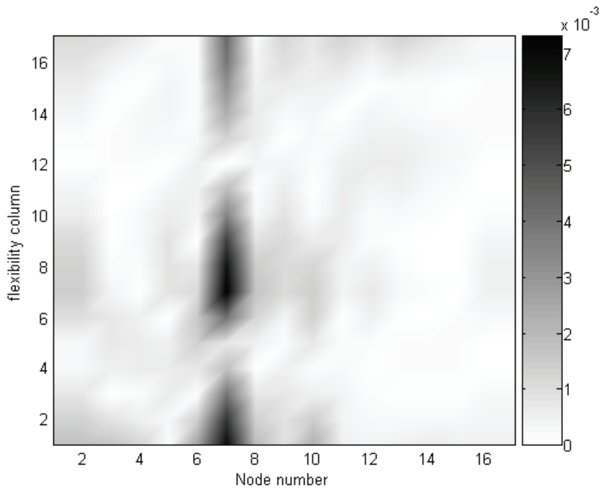

Fig. 15. The curvature of flexibility shapes: case A3

Both linear and cubic GSM were implemented on experimental flexibility shapes. Figures 16 to 18 show the results.

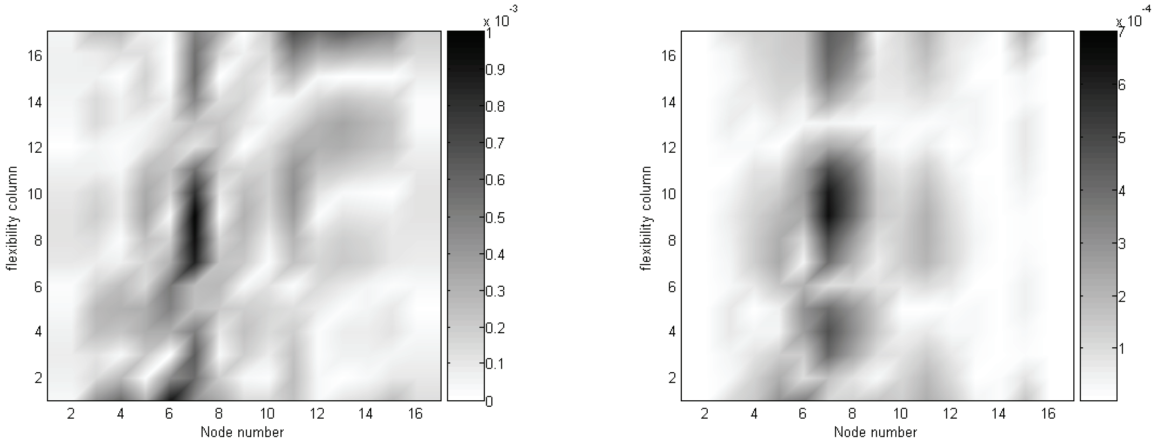

Fig. 16. Damage localization in case A1 via linear GSM (left) and cubic GSM (right)

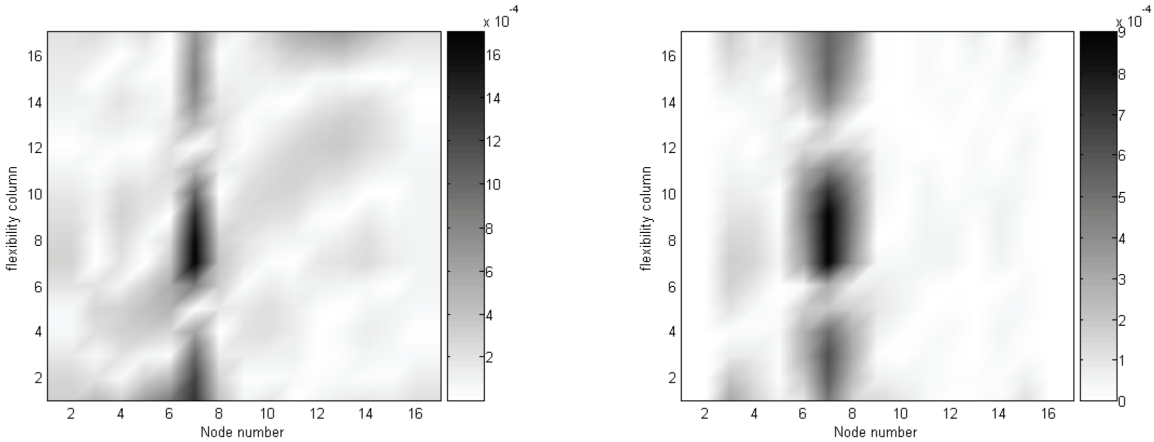

Fig. 17. Damage localization in case A2 via linear GSM (left) and cubic GSM (right)

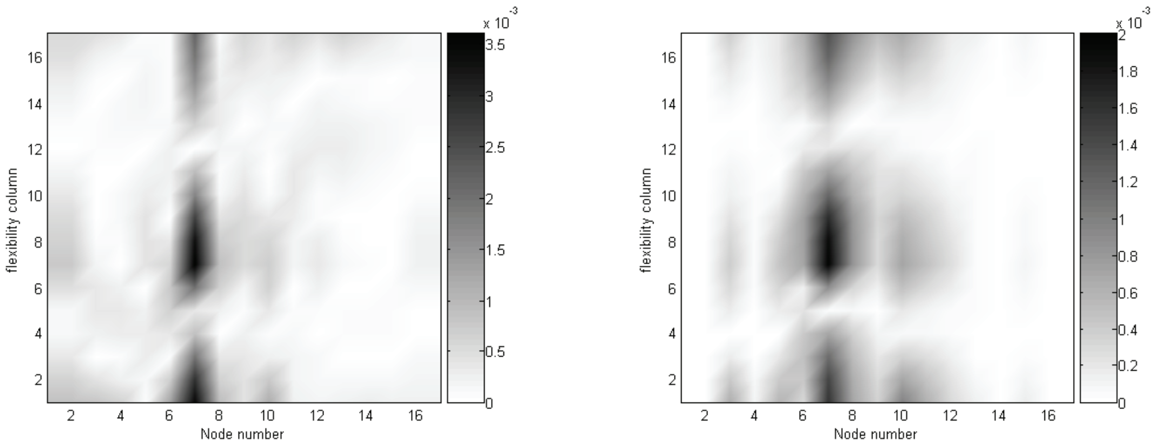

Fig. 18. Damage localization in case A3 via linear GSM (left) and cubic GSM (right)

The results show that the damage location has been well localized by both methods. Cubic GSM seems to work better, especially for the less severe damage case A1. As can be seen from Figs. 16 to 18, flexibility columns 4, 5, 13, 14 are less damage indicative. The reason is explained in the numerical case study in terms of sensitivity of damage indication to location of damage. Calculating curvatures of the three extracted modes in the healthy case reveals that the damaged site, i.e. node 7, has a curvature near zero for the third mode. Figure 19 shows the dependency factors of flexibility shapes to mode shapes in terms of modal assurance criterion formulation [16]:

$$MAC\left(F(:,i),\varphi_j\right)=\frac{\left|\{F(:,i)\}^T\{\varphi_j\}\right|^2}{\left(\{F(:,i)\}^T\{F(:,i)\}\right)\left(\{\varphi_j\}^T\{\varphi_j\}\right)} \tag{7}$$

Where $F(:,i)$ and φ_j are i th flexibility shape and j th mode shape respectively. Higher MAC between a flexibility shape and a mode shape can be concluded as the major contribution of that mode shape in the flexibility shape.

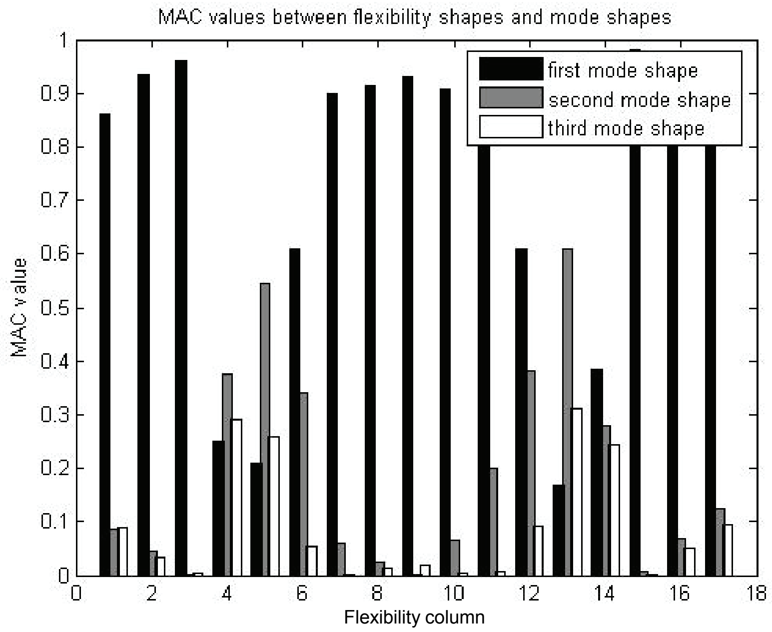

Fig. 19. MAC values between flexibility shapes and mode shapes

As expected, flexibility shape numbers 4,5,13 and 14 have a high contribution of the third mode whose curvature is near zero at the damaged zone. As a result, the aforementioned flexibility shapes are less indicative of damage near node 7.

Dynamically measured flexibility matrix can be used as a factor of the health state of the structures. Wherever the structural damage is, flexibility matrix characteristics will change as the health state of the structure changes. Figure 20 shows the trend of variation of various matrix norms due to making the structure more severely damaged in the experimental case study.

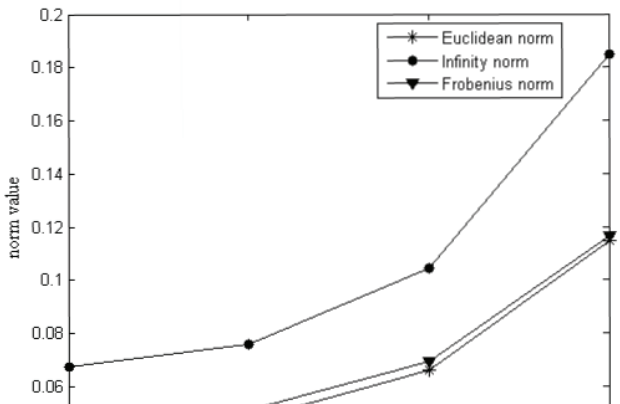

Fig. 20. Trend of flexibility matrix norm due to increasing damage extent

5. CONCLUSION

A method of structural damage localization has been proposed based on dynamically measured flexibility matrix. The method was successfully implemented on uniform beam structures, however, it can be easily extended for plate-like structures. The main advantage of the method is that it does not require theoretical model nor baseline information about the intact state of the structure. The DQM has been employed to increase the differentiation accuracy in calculating curvatures. Two kinds of linear and cubic GSM methods have also been implemented to locate the irregularity caused by damage in structural flexibility shapes. The method has been verified via numerical as well as experimental case studies. In both cases, the results are satisfactory and can be used for damage detection of real life structures.

REFERENCES

1. Sohn, H., Farrar, C. R., Hemez, F. M., Shunk, D. D., Stinemates, S. W., Nadler, B. R. & Czarnecki, J. J. (2004). A review of structural health monitoring literature from 1996-2001. Technical Report LA-13976-MS, Los Alamos National Laboratory Report.
2. Fritzen, C. P. & Jennewein, D. (1998). Damage detection based on model updating methods. *Mechanical Systems and Signal Processing*, Vol. 12, No. 1, pp. 163-186.
3. D'Ambrogio, W. & Zobel, P. B. (1994). Damage detection in truss structures using a direct updating technique. *Proc. 19th Int. Seminar for Modal Analysis on Tools for Noise and Vibration Analysis*, Katholieke Universiteit, Leuven, Belgium, Vol. 2, pp. 657–667.
4. Cawley, P. & Adams, R. D. (1979). The location of defects in structures from measurements of natural frequencies. *Journal of Strain Analysis*, Vol. 14, No. 2, pp. 49–57.
5. West, W. M. (1984). Illustration of the use of modal assurance criterion to detect structural changes in an orbiter test specimen. *Proceedings of the Air Force Conference on Aircraft Structural Integrity*, pp. 1–6.
6. Leiven, N. A. J. & Ewins, D. J. (1988). Spatial correlation of mode shapes, the Coordinate Modal Assurance Criterion (COMAC). *Proceedings of the Sixth International Modal Analysis Conference*, Vol. 1, pp. 690–695.
7. Lu, Q., Ren, G. & Zhao, Y. (2002). Multiple damage location with flexibility curvature and relative frequency change for beam structures. *Journal of Sound and Vibration*, Vol. 253, No. 5, pp. 1101-1114.
8. Pandy, A. K., Biswas, M. & Samman, M. M. (1991). Damage detection from changes in curvature mode shapes. *Journal of Sound and Vibration*, Vol. 145, No. 2, pp. 321–332.
9. Raghavendrachar, M. & Aktan, A. E. (1992). Flexibility of multi-reference impact testing for bridge diagnostics. *Journal of Structural Engineering, American Society of Civil Engineering*, Vol. 118, pp. 2186–2203.
10. Zhao, J. & Dewolf, J. T. (1999). Sensitivity study for vibrational parameters used in damage detection. *Journal of Structural Engineering, American Society of Civil Engineering*, Vol. 125, No. 4, pp. 410–416.
11. Pandy, A. K. & Biswas, M. (1994). Damage detection in structures using changes in flexibility. *Journal of Sound and Vibration*, Vol. 169, No. 1, pp. 3–17.
12. Zhang, Z. & Aktan, A. E. (1998). Application of modal flexibility and its derivatives in structural identification. *Research in Nondestructive Evaluation*, Vol. 10, No. 1, pp. 43–61.
13. Topole, K. (1997). Damage Evaluation via Flexibility Formulation. Smart Systems for Bridges, Structures, and Highways. *Proceedings of SPIE*, Vol. 3, pp. 145–154.
14. Shu, C. (2000). *Differential quadrature and its application in engineering*. Springer-Verlag, London.
15. Yoon, M. K., Heider, D., Gillespie, J. W., Ratcliffe, C. P. & Crane, R. M. (1994). Local damage detection using the two-dimensional gapped smoothing method. *Journal of Sound and Vibration*, Vol. 279, pp. 119-139.
16. Salehi, M. & Ziaei-Rad, S. (2007). Ground vibration test (GVT) and correlation analysis of an aircraft structure model. *Iranian Journal of Science & Technology, Transaction B: Engineering*, Vol. 31, No. B1, pp. 65-80.