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A NOVEL APPROACH TO OPTIMIZE GRINDING CIRCUITS- MODELLING STRATEGY TO MONITOR BALL MILL PARTICLE SIZE DISTRIBUTION DATA AT LAKAN PLANT[*]

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Abstract– Online monitoring and control of size distribution in any grinding circuit is a vital task in mineral processing plants. Flotation process efficiency depends on the size of feed materials and also fraction of the fine particles e.g. -38 microns. This paper presents a new approach to monitor the size distribution of Lakan Pb/Zn flotation circuit in Iran. A software monitoring size distribution has been developed, based on two main parameters named "n" which is reduction ratio in the mill and "b" which comes from Guadin equation, i.e. $W = aD^b$. "n" depends on mill characters and feed function while "b" depends on ore, instruments, and other properties. Liberation and size fraction studies showed that the d_{80} of ground material should be -74 microns and weight of passing 38 microns materials should be 35-40%, which is the optimum condition for the flotation recovery. Finally, results obtained from ball mill circuit for "n" and "b" indicated that these parameters were equal to "$n_{Ball\ Mill}$=80-87" and "$b_{Ball\ Mill}$=0.84-1.1" at optimum condition. Under this condition, flotation recovery increases up to 75%, which is practically confirmed in Lakan flotation plant.

Keywords– Monitoring, modeling, optimization, size distribution, Ball Mill, Lakan

1. INTRODUCTION

Computer simulation is a powerful and cost-effective tool for the design, analysis, and optimization of mineral processing unit operation [1, 2]. Almost all of the applicable models are strongly nonlinear and are not usually amenable to straightforward mathematical solution, nor are they always very convenient for easy computation using calculators or spreadsheets [3]. Quantitative modeling techniques and methods are an important approach for the development of process engineering and mineral processing [4, 5]. Models in mineral processing are difficult to develop because of the complexity of the unit operations used in virtually all mineral recovery systems. The major problem (among these difficulties) is the fact that the feed material is variably a particulate solid. Many conventional mathematical modeling techniques commonly used for processing equipment have limited their application to particulate systems and the models for most unit operations in mineral processing have unique features [6].

A flexible and powerful monitoring, on-line control and dynamic simulation approach for optimization of grinding circuits is being developed in some mineral processing plants in Iran. The application of dynamic simulation approach can help in the understanding of the complex and nonlinear behaviours and also dynamic interactions in grinding circuits. Monitoring mill data and dynamic simulation can be used to test ‘‘what-ifs’’ in grinding process operations such as circuit response to

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variations in feed and unit operation characteristics. This is a cheap and effective means of investigating circuit optimization without the risk of possible damage to the operating units or production of a large amount of unwanted products during a physical optimization process [7].

With the recent progress in on-line measurement of mineral processing, there are an increasing number of mineral processing variables that can be measured on-line in real time [8]. Real time dynamic computer simulation has been a powerful tool not only in traditional high-tech aerospace and military industries, but also in other areas such as automotive, steel making and chemical processing industries. However, until very recently, there has been limited practical application of the dynamic simulation in most of the mineral processing sectors, instead of relying on pilot plant studies and steady-state flowsheet simulation for plant design, equipment dimensioning and pre-control optimization [7].

Many simulation packages and techniques already exist for flowsheet simulation in mineral processing industries. They have been widely and successfully used for plant design, capacity planning (equipment sizing), circuit optimization, problem diagnosis and costing purposes. However, most of these existing simulation packages are based on steady state analysis (for instance; METSIM, USIM PAC, Limn and JKSimMet) and may utilize empirical models of limited generality for the individual unit operations [7]. Grinding, particularly at finer sizes to achieve the required degree of the liberation or specific surface, is a critical unit operation in terms of energy consumption and process optimization. Nowadays, considering the significant developments made in computer hardware and software, using simulation programs to optimize design and operation of various ore treatment plants, could be very beneficial to mineral processing engineers. Currently, there are some steady-state simulators that can be run under DOSTM or WINDOWSTM environments such as BMCS, MODSIMTM and JKSimMetTM [9]. The simulator allows the on-line modification of these parameters, and thereby evaluation of their effect on mill performance [10].

Mathematical modelling and computer simulation of the crushing and grinding process, particularly in the mineral industry, has been a very attractive research field for several decades. Most engineers use Microsoft Office Excel as the spreadsheet, because of its popularity, computational and graphical capabilities. The researcher has also implemented a mathematical model of crushing and grinding circuit flowsheet in this environment. This is due to the modelling, optimization and control methods, and selection to the circuits of crushing and milling. This progress has greatly improved the opportunities for more advanced control techniques by using total contour lines/curves response drawing (as followed in this research). These curves can be regarded as responses which can be selected to the effective factors. This approach can be easily employed in the monitoring of data and on-line control, to size reduction and manual flotation recovery. Sometimes there is not enough time to change the size reduction parameters (e.g. ball mill) to optimize the output size distribution. This means that the behaviours of size reduction equipment are a function of time. Therefore, studying the milling behaviour and monitoring the effective parameters are very important to keep the optimum condition in practice.

a) Problem statement

Understanding the complexities and effective parameters in steady state of a grinding circuit helps the control and management of operating costs. The major aims of this research work were:

i) Proposing a method for simple on-line control of size reduction by mathematical modeling.

ii) To assist operators to produce proper size distribution for increasing flotation efficiency in Lakan Pb/Zn plant. Hence, in this research, Lakan plant flowsheet and its classification equipment were employed, using Excel spreadsheet software. This model was developed by using data derived from simultaneous changes in reduction rate in the crushing and grinding circuit. Finally, it was concluded that the modelling and optimizing data in this technique will give a better chance (to operators) for achievement of the right size distribution and selecting more effective equipment, optimization process parameters and easy management of the plant.

2. APPROACH TO THE PROBLEM

a) The size reduction circuit in Lakan plant

Lakan mineral processing plant produces Lead and Zinc sulfide concentrates by flotation process. This plant is located near Lakan village, 27km east of Emarat Mine and 40km west of Khomeyn city in Markazi Province of Iran. This plant operates 335 days per year, three shifts per day with 71.8% recovery. Almost 350-400 ton/day of feed is provided from the Emarat Mine for this plant.

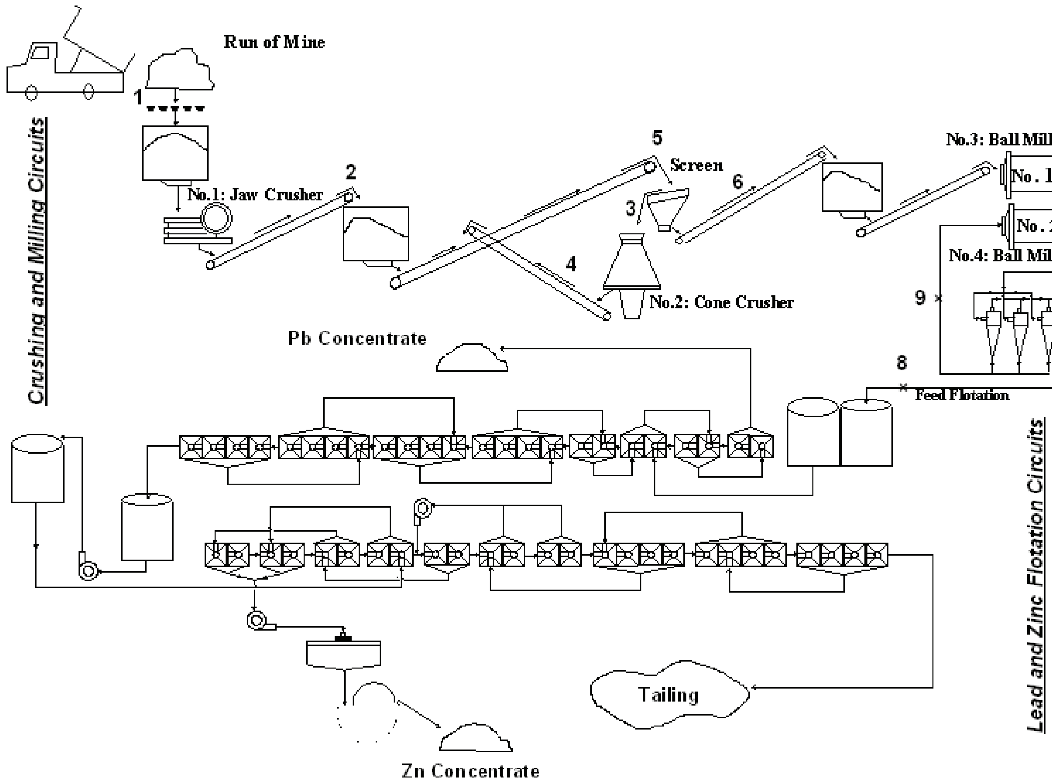

Fig. 1. The flowsheet of Pb/Zn processing in Lakan

The mineralization of the deposit contains two zones. In the Sulfide zone the main minerals are galena and sphalerite with an average grade of 2.1% and 5.5% for Pb and Zn, respectively. In the oxide zone the main minerals are cerusite and smithsonite with an average grade 0.5%Pb and 2.1%Zn. The plant feed is provided from the sulfide zone and recovers only sulfide minerals (galena and sphalerite). Qualitative and quantitative analysis of feed samples using XRF and XRD are shown in Table 1.

Table 1. XRF and XRD analysis of the sample from Lakan plant feed

XRF:	Zn: 5.8 %	Fe_2O_3: 6.7%	SiO_2: 25.5%	Al_2O_3: 2.2%	SO_3: 12.3%
	Pb: 2.1 %	TiO_2: 0.36%	CaO: 12.8%	MgO: 5.3%	L.O.I: 26.9%
XRD:	Barite, Dolomite, Quartz, Smithsonite, Cerossite, Geotite, Hematite, Sphalerite,Galena				

At first, this plant was designed to have one jaw crusher, one cone crusher, two ball mills (wet system), and six hydrocyclones for separating fine particles (flotation feed). Figure 1 shows the flowsheet of the crushing, grinding and flotation circuits in the Lakan plant. Changing the feed source from Lakan to the Emarat mine caused difficulties in providing a suitable size fraction for the flotation feed. Two ball mills are prior to flotation, one of which i.e. No. 2, was employed for simulation of the grinding circuit in this work.

b) Definition of problems in milling and flotation

The main problems of this plant could be classified as:

a. Lack of microscopic and degree of liberation studies on the flotation feed material.

b. High rate of fine and coarse particles production in mills and hydrocyclones.

c. Instability of product size distribution in classification and mills.

Low recovery of flotation process.

3. EXPERIMENTAL VERIFICATION

a) Sampling and characterization studies

A bulk of 350kg from the outlet of the jaw crusher was sampled for this study. After dividing by riffle, 5kg was finally sampled for grinding. The ground sample was then screened by using sieves apertures 1651, 208, 147, 124, 74 and 38 microns. The liberation study of the screened samples was carried out using Eq. (1) [11].

$$\begin{cases} D_L = \dfrac{1}{1+0.5(N_H + N_A.N_H)} = \dfrac{1}{1+0.5(N_H + N_{A.H})} \\ N_A = \dfrac{r}{k}, N_H = \dfrac{n_m}{n_L}, N_{A.H} = \dfrac{r.n_m}{k.n_L} \\ r = (0-100)\%, k = (0-100)\%, k \geq r \Rightarrow N_A = [0-1] \\ n_m = [0-\infty), n_L = [0-\infty) \Rightarrow N_H = [0-\infty) \end{cases} \tag{1}$$

where,

D_L : Degree of Liberation (%)

k : The lower degree of liberation acceptable in concentrate section (%)

r : The upper degree of liberation acceptable in tailing section (%)

n_L : Number of liberated mineral particles

n_m : Number of locked mineral particles

Microscopic and size distribution study of the samples showed that galena and sphalerite were liberated at size ~86 microns and 74 microns respectively, in which $k = 80\%$, and $r = 10\%$. It was accordingly concluded that d_{80}=74 microns was a suitable size for flotation feed. Results of this study are presented in Fig. 2, in which the rate of liberation of galena is greater than sphalerite and approximately in -74 microns, more than 80% of both minerals were liberated. Sieve analysis of flotation feed showed that it contains 29% of fine particles as slime. It was also shown that 34.5% of Zinc and 55% of Lead accumulated in the fine section (Table 2 and Figure 3). It is therefore considered that the grades of Zn and Pb in the flotation feed are 6.4% and 1.9% respectively. According to Fig. 3, 80% of cumulative mass in the flotation feed passed almost from -150 microns and the coarsest size particle was about 1.5mm. Since the optimum particle size which is accepted as liberated particle is 74 microns, 150 microns is not suitable and should be ground down to the desired size, i.e. 74 microns. Other than the liberation size, a particular size distribution was also needed. This size distribution should be as follows:

a: 98% -105 microns

b: 35-40% -37 microns

c: d_{80}=74 microns

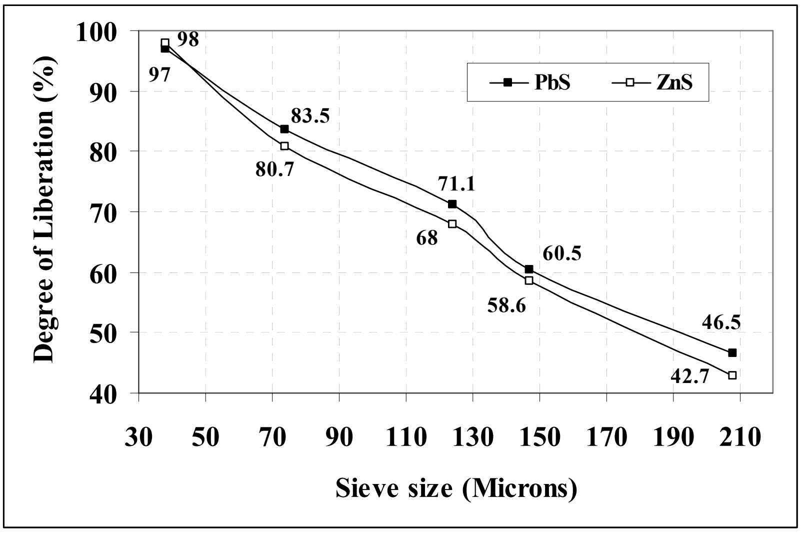

Fig. 2. Variation of degree of liberation versus size for PbS and ZnS

Table 2. Chemical analysis of different fraction of Lakan flotation feed

Size (microns)	remained on screen (%)	cum. passed mass (%)	grade (%)		metal cum. passed mass (%)	
			Zn	Pb	Zn	Pb
1651	-	100	-	-	100	100
208	6.5	93.5	3.89	1.5	96	95
147	16.1	77.4	3.02	1.16	88.4	85.2
124	17.2	60.2	5.36	1	73.9	76.2
74	8.6	51.6	5.86	1.36	66	70.1
38	22.6	29	8.91	1.31	34.3	54.7
-38	29		7.52	3.62		

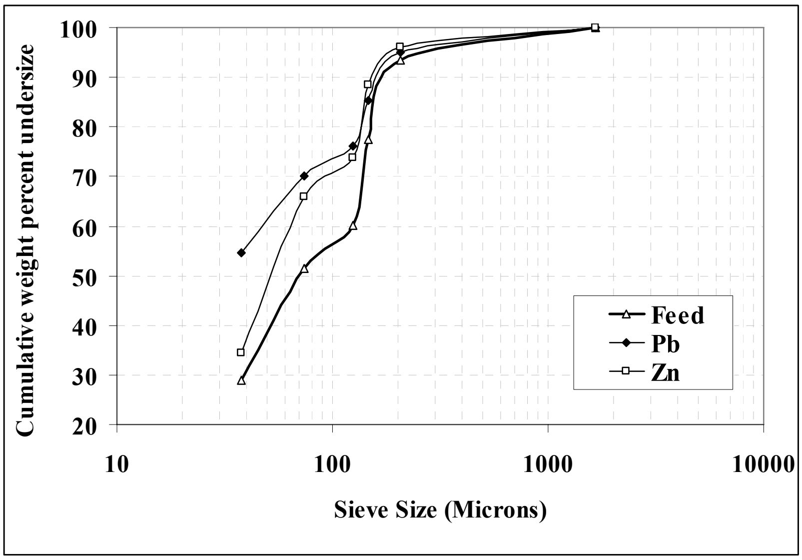

Fig. 3. Feed and Pb/Zn metals cumulative passed mass versus size in flotation feed

Tailing of the flotation process is deposited in the tailing dam. Microscopic and atomic absorption analysis of samples from the tailing dam showed some of the liberated galena and sphalerite dispatched to the tailing dam, showing an incomplete flotation of Zn and Pb particles. These particles were coarse and locked minerals. Figure 4 shows the microscopic pattern of the samples in the tailing. Some of these particles are coarser than 200 microns and some are liberated too. Figure 5 shows the variation of flotation recovery over 18 months. This confirms the low recovery due to fine particles, locked and coarse particles. Some fluctuations were observed in lead and zinc flotation recoveries in which the average flotation recovery of lead and zinc sulfides were 50.0% and 69.1% respectively.

It was concluded that there were some problems in the grinding circuit, feeding the flotation process. Bearing in mind the above problem, it was decided to model and monitor the crushing and milling process. The model was developed using excel software [12, 2].

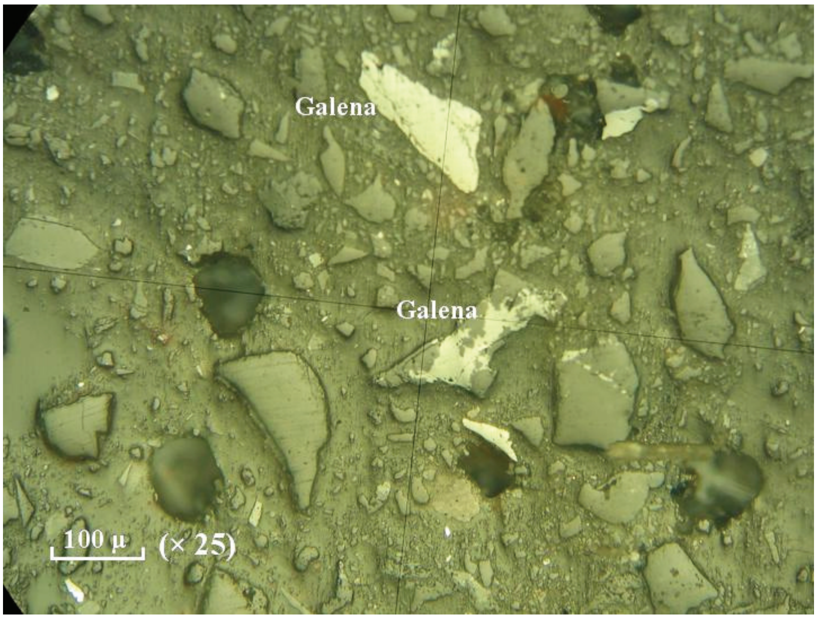

Fig. 4. Microscopic image of mineral's texture in a sample of tailing dam

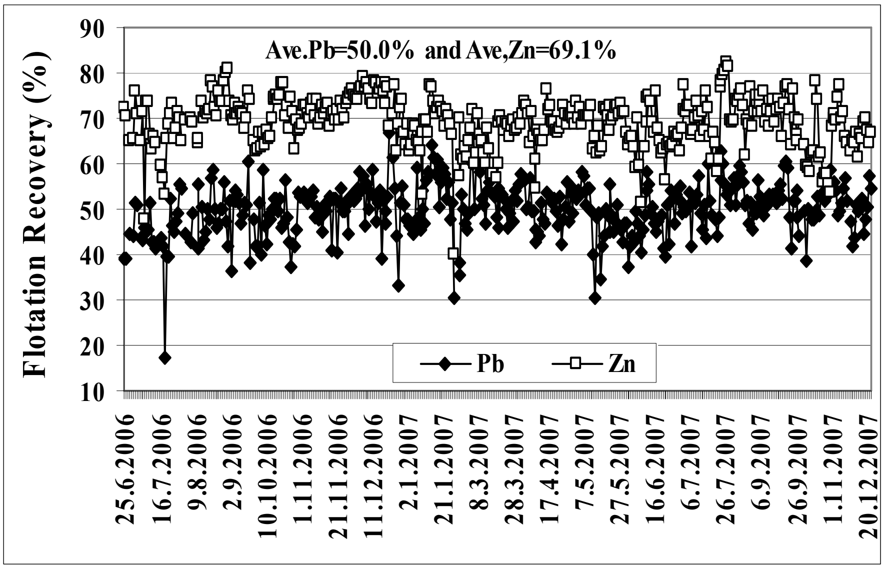

Fig. 5. Recovery of lead and zinc flotation from July 2006 to November 2007

b) Modelling strategy

To fit the curves, a model similar to Gaudin model was employed [13, 14]. This model is a power function as shown in Eq. (2):

$$w = aD^b \tag{2}$$

where W is cumulative percentage passed mass, D is the size of particle or screen aperture, and "a" and "b" are constant. By re-writing the above equation to logarithmic function, it finally ends in Eq. (3):

$$\text{LogW=Loga+bLogD} \tag{3}$$

If LogW=Y, Loga=A and LogD=X, then Y=A+bX

In the new function, "b" is the slope of the line in log×log curve scale. In another way, "b" Slope Grinding Function (SGF), is a function of some effective parameters which will be discussed later. Gaudin model is applied for D_{80} and fine particle slimes (-38 microns), as the most important parameters. If W is to be 80%, then D is D_{80} and:

$$0.8 = a.d^b_{80-output} \Rightarrow a = \frac{0.8}{d^b_{80-output}} \tag{4}$$

If "n" is to be mill reduction ratio, then:

$$n = \frac{D_{80-input}}{d_{80-output}} \Rightarrow d_{80-out} = \frac{D_{80-input}}{n} \tag{5}$$

By compiling Eqs. (4) and (5):

$$a = \frac{0.8}{\left(\frac{D_{80-input}}{n}\right)^b} \tag{6}$$

and finally:

$$w = 0.8\left(\frac{n \times D}{D_{80-input}}\right)^b \tag{7}$$

where, $D_{80-input}$ is the size of feed particles of any crushers or mills, which presents that 80% of cumulative passed mass from this size, and "b" depends on the type of crushing and grinding sets, properties of minerals/rocks and other factors such as:

b= f (type of equipment and milling process, capacity, size distribution, properties of rocks /minerals mechanic, fractures, moisture, degree of liberation, hardness, targets and etc.) (8)

Then, end size, P_{Final}, can be defined as:

$$P_{Final} = f(D_{Feed}, n_i, b_i) \tag{9}$$

where, "b_i" refers to each parameter of grinding and size separation. Table 3 shows some used limits of "b" and "n" in Lakan plant in some working periods.

Table 3. The values of "b" and "n" for different equipment of Lakan plant

equipment	B	N
jaw crusher	0.3-0.75	2-6
cone crusher	0.4-0.85	2-8
ball mills	0.2-0.75 (wet)	30-120

In this work, Eq. (6) is a base model to simulate and optimize parameters (n and b) in Lakan milling processes. In the following and figures, values of 1, 2, 3 and 4 of "n" and "b" refer to the jaw crusher, cone crusher, ball mill No. 1 and ball mill No. 2., as it was presented in Fig. 1.

c) Validation of model

According to the obtained final size limit, using $w = aD^b$ and plotting cumulative curves in each circuit in the Excel spreadsheet, each section of circuits was modelled and optimized. Finally, the best condition was selected for this plant. In each section of the models, by changing "n" and "b" and also based on the resulted size limit, it was optimized under a different scenario:

1. Without any change in flowsheet: After modelling the plant flowsheet and by changing "n" and "b" in the jaw crusher, cone crusher and two ball mills, no ideal results were obtained, because if "n" and "b" were fixed in ball mill No.2 (working condition plant at the present time), to decrease fine particle and yielding aim size limit, "n" should be very small (almost five) and "b" should be less than 0.1 in ball mill No.1 or "b" to be more than 1.7. This is just a theoretical condition and is not practical. Figure 6 shows -38 microns particles fraction in the underflow of hydrocyclone (circuit 9), which is the circulation load of ball mill No.2, to control fine particles.

On the other hand, it is impossible to reach a suitable size distribution with an acceptable limit of "b" for ball mill No. 1. So, in the modified flowsheet, there are almost more than 40% fine particles which contain considerable amounts of Pb and Zn metals and coarse particles. Therefore, the condition of the system should be changed to optimize and produce a suitable size distribution by comparing the target curve for the flotation process.

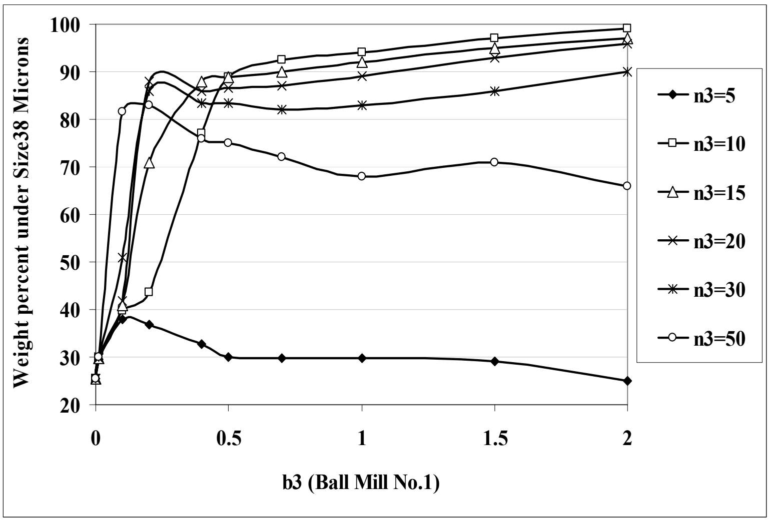

Fig. 6. Percentage of -38 microns versus the variation of "n_3" and "b_3"in hydrocyclone underflow

2. Changing the aperture of the screen and size separation of the hydrocyclone: In this section, by changing the screen aperture from 8 to 9.5, 11.3 and 13.4 mm, different types of models were optimized. The final results of modelling were as follows:

a) By 9.5 mm: it was resulted n_3=70-80, n_4=40, b_3=1.5, b_4=0.5, the percentage of fine particles was 34% and final production was equal to d_{80}=63 microns.
b) By 11.3 and 13.4mm: it did not yield a good result, and had the same results as the model produced for 9.5mm.
c) Changing the hydrocyclone cut size to 125 microns. By considering the same rate of grinding; "n=10" for two ball mills and accepting actual "b", the model response was good, but it was very sensitive, because d_{80} of final production and the percentage of fine particles changed by very small changes in "n".

Considering the above changes, and analyzing the results, it is stipulated that the responses were quite good. A new case of simultaneous changes in screen aperture to 9.5 and 13.4mm and hydrocyclone cut size to 90 and 125 microns, a new model was developed which was much better for optimization. If the crushing circuit feed size is to be maximum coarse and the ball mill No.2 works in actual condition, the best size distribution curve will be obtained by changing "n_3" and "b_3" (Fig. 7).

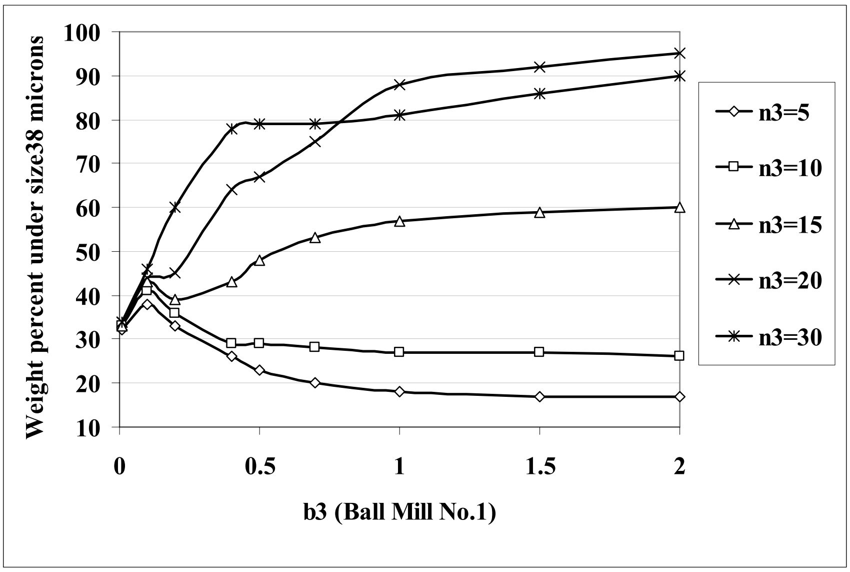

Fig. 7. The percent of -38 microns versus the variation of "n_3" and "b_3"in hydrocyclone underflow

Figure 7 shows the results of -38 microns percentage in hydrocyclone underflow (circuit 9), which is reported into the ball mill No.2 to control fine particles in the ball mill No.1, by changing "n_3" and "b_3". In this condition, the weight of fine particles is very high and by increasing "b" in ball mill No.2 it is improved, but not completely. It was therefore concluded that this condition of model would not be appropriately applicable.

3. Elimination of the ball mill No.1 in flowsheet: According to the previous results, changing "n" and "b", "n_3" and "b_3" parameters is suitable when they are very small. This means that ball mill No. 1 should turn into rod mill or be removed. By increasing "b_3" in ball mill No.1 the performance of ball mill No. 2 becomes stable, and of course it should be noted that this state is practically impossible. Thus in this section, ball mill No. 1 is accordingly removed and by only one ball mill the flowsheet is modelled and optimized. Thus, it seems that the production of fine particles decreases and a considerable amount of the energy is saved. In this model the product of the grinding circuits is conducted into the hydrocyclone and its underflow reports to the ball mill No. 2. The condition of the crushing circuits shows an identical situation with the plant (actual) condition in the production of coarse particles under the optimum state (13.4mm for sieve aperture, n_1=2 and b_1=0.73 for jaw crusher and n_2=6 and b_2=0.84 for cone crusher) and hydrocyclone cut size should be 105 microns for the flowsheet output analysis. Figure 8 indicates that changing "n_1" and "n_2" from the jaw and cone crushers is not effective on the fine particles production, but it is significant to produce enough coarse feed for ball mill No.2.

After removing ball mill No.1 and by changing "b_4" and "n_4" in the new model, some parameters such as d_{80} of ball mill No. 2 product, percentage of -38 microns particles and circulation load percent were calculated and are shown in Table 4. All the results were analyzed and the best on-line control path was determined for engineers and operators. There are several methods to show curves and results analysis of data modelling or laboratory tests. Some of them are 2D as contour curves (by two parameters) or 3D as surface. By employing the above methods the required results and plotting the contour lines (2D) and their surface response (3D), analysis sensitivity and on-line control management on variation data are easily and carefully achieved. In Table 4 and Figs. 9 to 11, the responses of the model and their curves are shown as 2D for hydrocyclone overflow.

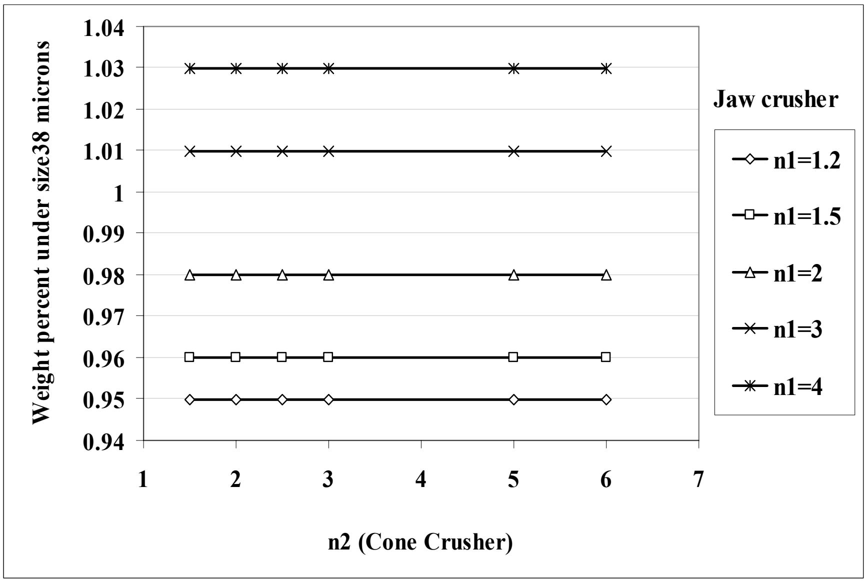

Fig. 8. Percentages of -38 microns by changing "n_1" and "n_2" in crushing circuit

The contour curves are confined by "n" and "b" which were selected for the production of d_{80}=74 microns and production of fine particles, less than 35-40%. This is the best method (path) to define the limitation of "n" and "b" in the model to be applied for the Lakan plant. All the data in Table 4 were rounded. Figures 9, 10 and 11 show contour lines of d_{80} and -38 microns and circulation load percents, using Surfer 07 software and Kriging model. If one response contour line is selected, concerning two dependant parameters, the amount of the response is to be constant.

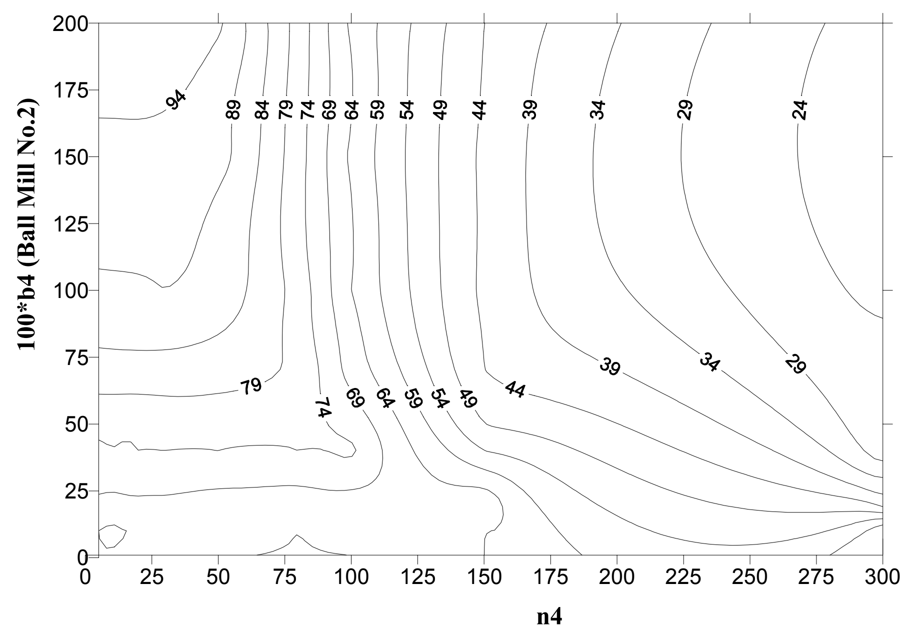

Fig. 9. Extrapolation and estimation of amount of d_{80} by changing "n_4" and "b_4" in Ball Mill No. 2

Table 4. Results of model for final production of flotation feed by changing "b_4" and "n_4" in Ball Mill No. 2

Ending size production of d_{80} from flowsheet by changing "n_4" and "b_4"										
b_4	n_4=5	n_4=10	n_4=15	n_4=20	n_4=30	n_4=50	n_4=80	n_4=100	n_4=150	n_4=300
0.01	65	64	66	66	66	64	64	64	64	64
0.1	64	63	64	64	64	65	64	65	64	64
0.2	68	67	68	68	68	67	66	67	68	42
0.4	73	74	73	74	74	74	74	75	54	26
0.5	75	76	75	75	76	76	75	72	49	26
0.7	82	82	82	82	82	81	78	67	44	26
1	88	88	88	88	89	87	77	64	43	23
1.5	92	92	92	93	92	91	76	63	43	22
2	97	96	97	96	96	95	77	63	44	22
Percentage of under size 38 μ in ending production by changing "n_4" and "b_4"										
b_4	n_4=5	n_4=10	n_4=15	n_4=20	n_4=30	n_4=50	n_4=80	n_4=100	n_4=150	n_4=300
0.01	61	61	61	61	61	61	61	61	61	61
0.1	61	61	61	61	61	62	62	62	62	62
0.2	58	58	58	58	59	58	58	58	58	76
0.4	51	51	51	51	51	52	51	51	64	98
0.5	48	48	48	48	48	47	48	51	66	98
0.7	41	41	41	41	41	41	41	49	69	98
1	32	32	32	32	32	32	35	44	69	98
1.5	20	20	20	20	20	20	27	36	70	98
2	12	12	12	12	12	12	19	29	67	98
Percentage of Circulation load by changing "n_4" and "b_4"										
b_4	n_4=5	n_4=10	n_4=15	n_4=20	n_4=30	n_4=50	n_4=80	n_4=100	n_4=150	n_4=300
0.01	126	125	125	124	124	123	122	122	121	121
0.1	161	148	141	137	132	126	120	118	114	107
0.2	194	172	161	154	143	130	120	116	108	98
0.4	278	231	205	186	166	141	121	113	98	98
0.5	316	253	226	205	178	148	122	111	98	98
0.7	382	316	260	239	202	160	125	107	98	98
1	425	359	316	278	229	175	126	102	98	98
1.5	494	429	382	338	279	206	128	98	98	98
2	530	490	427	397	331	235	130	98	98	98

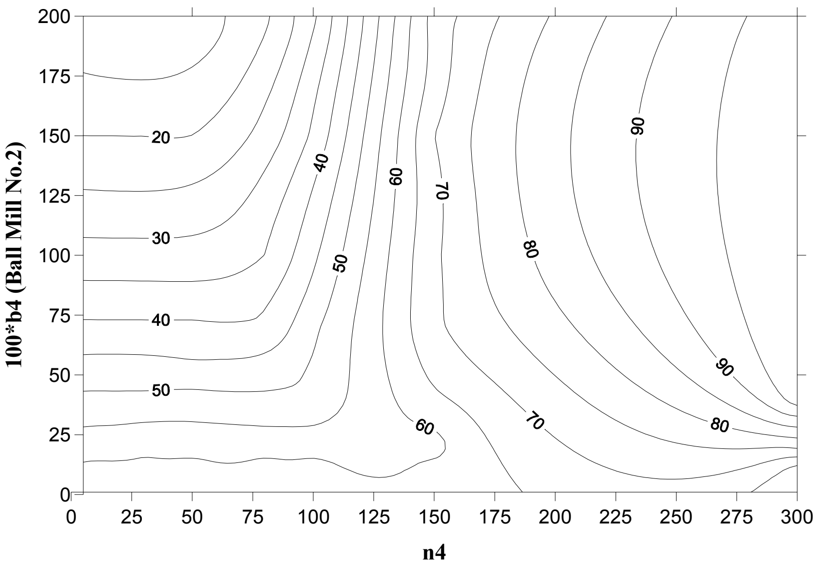

Fig. 10. Extrapolation and estimation of -38 microns by changing "n_4" and "b_4" in Ball Mill No. 2

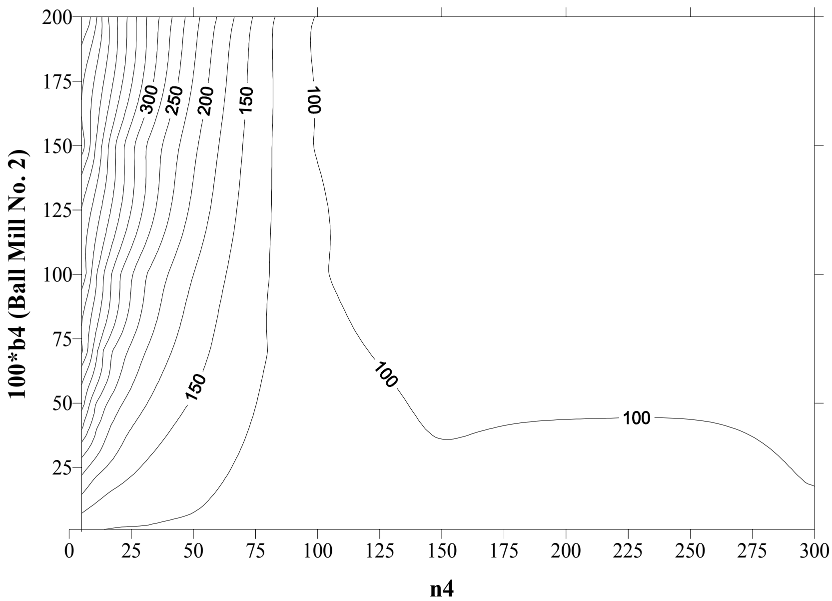

Fig. 11. Extrapolation and estimation of circulation load by changing "n_4" and "b_4" in Ball Mill No. 2

d) Error correction of model

Because of some errors produced in the interpolation of data, two Eqs. (10) and (11, were used to check and respond to the amendments. Data interpolation by power function ($W = aD^b$) is not a very exact method and R square value is less than 1 (R^2 <1). According to Eq. (7):

$$ifD = d_{80-out} \Rightarrow w = 0.8\left(\frac{n \times D}{D_{80-in}}\right)^b = 0.8\left(\frac{D}{d_{80-out}}\right)^b = 0.8 \Rightarrow w = 80\% \tag{10}$$

$$ifD = 38\mu, d_{80-out} = 74\mu \Rightarrow w = 0.8(0.514)^{b_{BallMill}} \Rightarrow b_{BallMill} = \frac{Ln(\frac{w}{0.8})}{Ln0.514} \tag{11}$$

where $b_{BallMill}$ comes from Eq. (10) and is the response of the impact of d_{80} and fine particles curves. In Table 5 index "f" and its relationship with fine particles percentage are defined. Index "f" was defined as Eq. (12).

$$b_{BallMill} = (1+f)b_{Curve} \tag{12}$$

In this condition, by sampling from the input and output of ball mill No. 2 and plotting the cumulative passing curve in Log×Log scale, n_4 (Eq. (13)) and b_4 (slop of line) are accordingly and easily calculated. By correcting "b", using Eq. (12) and Table 5 and Fig. 12, and transferring it into Fig. 13 (d_{80}=74 microns and 35-40% fine particles), the behaviour of the ball mill and its variation state could be analyzed. Finally, applying this methodology, analysis of ball mill mathematical modelling and optimizing, monitoring and on-line control would be facilitated, and management of flotation recovery would be more logical. It should be noted that there are some effective parameters to expand "b" function for better control, which is currently investigated.

$$n_4 = \frac{D_{80-in}}{D_{80-out}} \tag{13}$$

Table 5. Index "f" and its relationship with -38 microns percent (fine particles)

-38 microns (%)	F
20	0.014
25	0.014
30	0.049
35	0.113
40	0.183
45	0.282
50	0.362

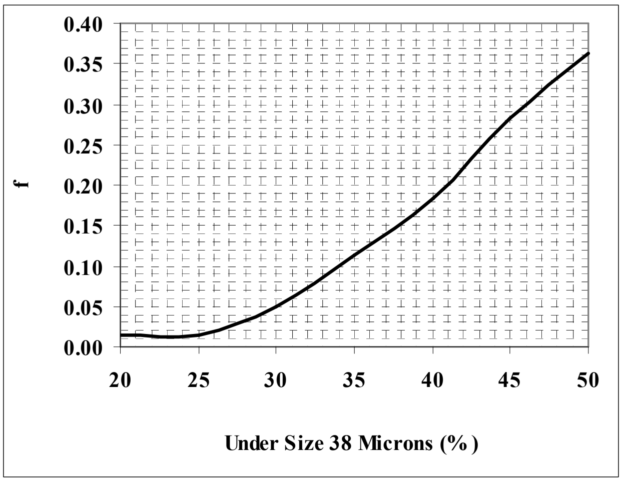

Fig. 12. Curve of index "f" and its relationship with -38 microns percent

4. RESULTS AND DISCUSSION

For optimization of the suitable particle size distribution, the reduction ratio-slope in grinding function model (or "n-b" model) was employed. The n-b model yielded accurate results and was easy access. Clearly, the n-b model optimization approach provides a sufficient approximation to the true optimal solution. Some advantages of the developed n-b model are:

• Simple complementing and programming of the model by only two parameters without modifying the existing model structure.

• Automatic searching of non-linear connection and finding the optimum condition between d_{80} and percentage of produced fine particles (-38 microns).

• Fast and simple monitoring, optimizing and control of grinding and crushing circuits.

The d_{80} was calculated in Fig. 9 by changing "n_4" and "b_4" for flotation feed. The rate of grinding in ball mill No. 2 was n_4=0-300 and limitation of "b" was b_4=0-2. In "liberation degree" studies, it was resulted that d_{80} should be 74 microns. Then, by selection of 74 microns contour line in Fig. 9, acceptable limits of "n_4" and "b_4" were estimated. By considering n_4=0-100 and d_{80}=74 microns, the rate of b_4 was 0.4. But considering n_4=80-86 and increasing b_4, this limit changed to 0.4-2. Thus ball mill No. 2 could present two behaviours in this situation. The optimum conditions predicted by the n-b model are as follows:

a) n_4=0-100 and b_4=0.4

b) n_4=80-86 and b_4=0.4-2

Sometimes, after changing feed characterization and other parameters, the function of ball mill is impressed. Likewise, the mineral engineer would like to bring this change back to the initial condition without plant shutdown. Sensitivity analysis of parameters to control ball mill is necessary for monitoring and dynamic on-line control. According to Fig. 9, at a low rate of grinding mill (n_4) and low rate of b_4, coarse particles in the output of the system for flotation feed (Fig. 1) is observed. If "n_4" is less than 70, by increasing the "b_4" parameter, the particles become coarser than 74 microns. If "n_4" is more than 70 and "b_4" increases in the system, the particle size of d_{80} decreases drastically. Also, when the "n_4" is more than 200 and the b_4 more than 1, d_{80} decreases drastically to less than 35 microns. Finally, when the rate of milling (n_4) is very high, b_4 should be changed to the lowest rate (less than 0.25).

Weight percentage of -38 microns (fine particles) is calculated in Fig. 10 by changing "n_4" and "b_4" for flotation feed. The target of this research was to produce 35-40% of fine particles. So, by selecting 35 and 40 contour lines in Fig. 10, "n_4" and "b_4" are estimated. By considering n_4=0-80, "b_4" would be in the limit of 0.74-1.1. In this limit, the rate of fine particle, production is to be 35-40%. It was found that in this limitation, by selecting "n_4" and increasing "b_4", the rate of fine particles will be decreased. The rate of "b_4" is more than 1.1; it decreases to less than 30% drastically. Thus, it was also found that in n_4=80-110 and b_4=1.1-2, the target was achieved. Meanwhile, for decreasing the rate of fine particle production in ball mill No. 2, if "n_4" is less than 80, "b_4" should be increased. Then, by understanding the practical condition of the ball mill, transferring it on the graphs and comparing it with the optimum point, one would easily be able to guide ball mill behaviour toward the optimum point.

By using these contour lines, sensitivity analysis of the percentage the fine particles is estimated. It is predictable that the high production of fine particles would be possible by using high values of "n_4" and low "b_4". Therefore, by accepting "n_4" less than 100 and "b_4" more than 0.7, fine particles less than 40% are then produced. But by increasing "n_4" more than 100, the responses increase drastically. Understanding the milling behaviour of fine particles production rate could be a good method to monitor and on-line control and their optimization (Fig. 10). Finally, according to the overlaying rate production of d_{80}=74 microns and percent of -38 microns particles less than 35-40%, it would be possible to specify the interactive area and some of the suitable points (Fig. 13). Figure 13 contains the entire expected conditions of d_{80} and fine particles (-38 microns) production by removing ball mill No. 1 (Fig. 1). In Fig. 13, the d_{80} contour lines and the contour lines of the fine particles were shown in solid and dash lines respectively. If d_{80}= 74 microns contour line and two contour lines of fine particles for 35%, 40%, intersect each other, "A", "B" (and limit between "A", "B" points) will be obtained. Finally, "n_4" and "b_4" are equal to 80-86 and 0.84-1.1 respectively. For modelling the circulation load in the ball mill, the optimum point of "n_4" and "b_4" are transferred to Fig. 11 and in the new condition, the circulation load becomes 120%-125%.

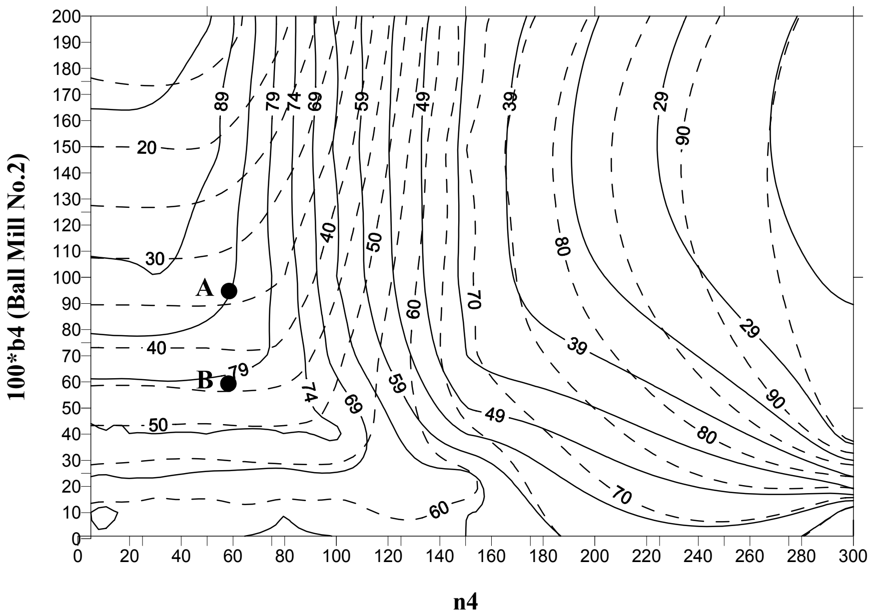

Fig. 13. Interaction of d_{80} and fine particle response contour lines, (——— d_{80}, --------- Fine particle)

5. CONCLUSION

This research work presents the development and use of modelling and monitoring methodology in milling process for on-line control and optimization of the size particles distribution in ball-end production milling. The milling model is software based, in which excel software was employed. This model applies continuous system, theoretic knowledge of technological processes, easy modelling and programming to control and/or to be able to correct errors of performed tests. All influencing factors, i.e.: reduction rate, "n" and slope grinding function, "b" were considered. The on-line monitoring system provides a new practical way to obtain suitable particle size distribution in the ball milling process. This modelling strategy was set by use of only "n" and "b" parameters, and using the optimization approach to solve the control of milling operations problems, with ball end size particle production milling. In this paper the results obtained from the proposed novel approach to optimize the grinding process, proves its effectiveness on the (flotation) feed production and its recoveries. The implication of the encouraging results obtained from the present approach is that such approach could be integrated in monitoring and on-line control of ball mill, with an intelligent manufacturing system for automated process planning and design based on new software. Since the n-b modeling approach can yield near-optimal solution, it could therefore be used for parameters selection monitoring of complex milling processes which require optimizing and easy control of many milling constraints. Integration of the proposed approach with a cumulative contour lines responses system will lead to a reduction in production cost and production time, flexibility in the control of milling parameter selection, and improvement of product quality.

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