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NUMERICAL ANALYSIS OF TURBULENT NATURAL CONVECTION IN SQUARE CAVITY USING LARGE-EDDY SIMULATION IN LATTICE BOLTZMANN METHOD*

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Abstract– In this paper the turbulent natural convection flow is investigated through Large Eddy Simulation (LES) turbulence model, which is applied in Lattice Boltzmann, and the results for high Rayleigh numbers between 10^6 and 10^9 are represented. Air with Pr=0.71 is considered in this study. In this investigation we have tried to present Large-eddy turbulence model by Lattice Boltzmann Method (LBM) with a clear and simple statement. The results confirm that this method is in acceptable agreement with other verifications of such a flow. The effects of increase in Rayleigh number are displayed on streamlines, isotherm counters, local Nusselt number and average Nusselt number.

Keywords– Turbulent natural convection, Large Eddy Simulation, Lattice Boltzmann Method

1. INTRODUCTION

The description of turbulence is a fundamental problem in flow engineering and theoretical research in fluid mechanics. Turbulent flows consist of large eddies as well as dissipating small structures and therefore cover a wide range of scales and energies. Numerous studies can be found in literature on turbulent flow [1-3]. Turbulent flows are modeled by various methods, of which the most traditional method is large-eddy [4, 5]. This model is applied in various applications such as analysis of geophysical phenomena in the atmosphere, oceans and magnetosphere and provides a starting point for modeling these phenomena. Also, the confinement of thermonuclear plasmas and superfluid and superconductive behavior of thin films have been investigated by this method [6-9].

However, turbulence research requires detailed information attained by numerical simulations and thus benefits from the development of new algorithms and the evolution of high performance computers. Therefore, developing a numerical method which presents the best compromise between the conflicting requirements of low computational cost and high numerical accuracy still remains an open problem. The lattice Boltzmann method has been applied extensively in the past two decades. As an effective and easy numerical method, it is utilized in simulating complex flow problems with different boundary conditions such as multi-phase flows, multi-component flows, particulate suspensions, turbulent flow and micro-flows [10-19]. Lattice Boltzmann Method demonstrates that it can be a powerful method for simulating turbulence flows [20-24]. The implementation of a LBM procedure is much easier for turbulence flows than that of traditional CFD methods. Meanwhile, it is more popular due to the balance between accuracy and efficiency. Yu et al. [25] considered the application of multiple-relaxation-time (MRT) lattice Boltzmann equation (LBE) for large-eddy simulation (LES) of turbulent flows. They demonstrated that

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MRT-LBE is a potentially viable tool for LES of turbulence. Fernandino et al. [26] investigated large eddy simulations of turbulent open duct flow performed using the lattice Boltzmann method (LBM) in conjunction with the Smagorinsky sub-grid scale (SGS) model, and found that the LBM simulation results are in good qualitative agreement with the experiments. Chen [27] proposed a novel and simple large-eddy-based lattice Boltzmann model to simulate two-dimensional turbulence. He showed that the model is efficient, stable and simple for two-dimensional turbulence simulation. The Lattice Boltzmann Method (LBM) with a forcing scheme is used to simulate homogeneous isotropic turbulence by Kareem et al. [28]. They found that the turbulence characteristics of the flow are similar to those obtained in studies by the spectral method, regardless of which model is used.

The main aim of this investigation is to present Large-eddy turbulence model by Lattice Boltzmann Method (LBM) with a clear and simple statement. Thus natural convection turbulence flow in a cavity is investigated in a wide range of Rayleigh numbers. First, LBM is considered briefly and then large-eddy is applied in LBM where the equations are expressed completely. Finally, the results of this study are compared with the past researches.

2. NUMERICAL METHOD

a) Problem statement

The geometry of the present problem is shown in Fig.1a. A two-dimensional enclosure with a height of H and width of W is displayed. The temperatures of the two sidewalls of the cavity are maintained at T_H and T_C, where T_C has been considered as the reference condition. The top and the bottom horizontal walls have been considered to be adiabatic i.e., non-conducting and impermeable to mass transfer. The density variation in the fluid is approximated by the standard Boussinesq model. The fluid is assumed to be Newtonian, incompressible and the laminar, whereas Prandtle number is equal to Pr=0.71. Also, it is assumed that Mach number is fixed at Ma=0.1.

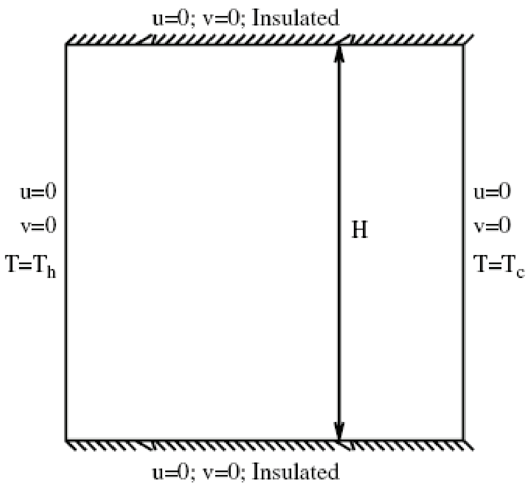


Fig. 1a. Geometry of the present study

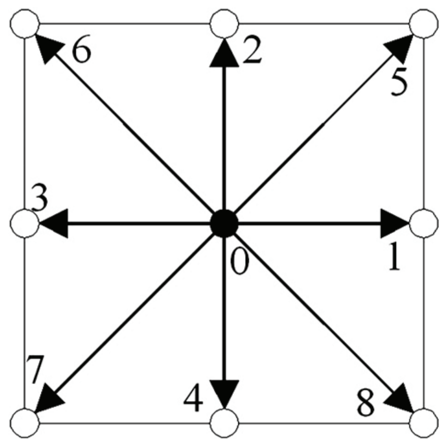


Fig. 1b. The discrete velocity vectors for D2Q9

b) Lattice Boltzmann method

For the incompressible flow, if the transport coefficients are independent of the temperature, the energy equation can be decoupled from the mass and momentum equations. For the incompressible thermal problem, *f* and *g* are two functions called flow distribution function and temperature distribution function respectively. These functions are utilized to obtain macroscopic characteristics of the flow like velocity, pressure, temperature and etc.

In this paper a square grid and D2Q9 model is used for both flow and temperature functions. By detachment of Navier-Stocks equations, governing equations for flow and temperature functions are as follows [29-32]:

For the flow field:

$$f_i\left(x+c_i\Delta t,t+\Delta t\right)-f_i\left(x,t\right)=\frac{1}{\tau_v}\left[f_i\left(x,t\right)-f_i^{eq}\left(x,t\right)\right]+\Delta t c_i F \tag{1}$$

For the temperature field:

$$g_i\left(x+c_i\Delta t,t+\Delta t\right)-g_i\left(x,t\right)=\frac{1}{\tau_c}\left[g_i\left(x,t\right)-g_i^{eq}\left(x,t\right)\right] \tag{2}$$

Standard D2Q9 for flow and temperature, and LBM method is used in this work where the discrete particle velocity vectors defined c_i , Δt denotes lattice time step, τ_v, τ_c is the relaxation time for the flow and temperature fields, respectively , f_i^{eq} , g_i^{eq} is the local equilibrium distribution function that has an appropriately prescribed functional dependence on the local hydrodynamic properties which are calculated with Eqs. (3) and (4) for the flow and temperature fields respectively [27], and F is an external force term.

$$f_i^{eq}\left(x,t\right)=\omega_i\rho\left[1+\frac{c_i u}{c_s^2}+\frac{1}{2}\frac{(c_i u)^2}{c_s^4}-\frac{1}{2}\frac{u u}{c_s^2}\right] \tag{3}$$

$$g_i^{eq}=\omega_i T\left[1+\frac{c_i u}{c_s^2}\right] \tag{4}$$

The discrete velocities, c_i, for the D2Q9 (Fig. 1b) are defined as follows:

$$c_i=\begin{cases} 0 & \text{i=0} \\ c\left(\cos[(i-1)\pi/2],\sin[(i-1)\pi/2]\right) & \text{i=1-4} \\ c\sqrt{2}\left(\cos[(i-5)\pi/2+\pi/4],\sin[(i-5)\pi/2+\pi/4]\right) & \text{i=5-8} \end{cases} \tag{5}$$

where ω_0=4/9, $\omega_{1–4}$=1/9, $\omega_{5–8}$=1/36 and $c=\sqrt{3RT_m}$ (to improve numerical stability, T_m is the mean value of temperature for the calculation of c.

Using a Chapman-Enskog expansion, the Navier-Stokes equations can be recovered with the described model. The kinematic viscosity υ and the thermal diffusivity α are then related to the relaxation times by

$$\vartheta=\left[\tau_v-\frac{1}{2}\right]c_s^2\Delta t \quad \text{and} \quad \alpha=\left[\tau_c-\frac{1}{2}\right]c_s^2\Delta t \tag{6}$$

where c_s is speed of sound and equal to $c/\sqrt{3}$.

In the simulation the Boussinesq approximation is applied to the buoyancy force term. In this case, the external force F appearing in Eq. (1) is given by:

$$F_i=3\omega_i g_y\beta\Delta T \tag{7}$$

where g_y, β and ΔT are gravitational acceleration, thermal expansion coefficient and temperature difference, respectively.

Finally, the macroscopic quantities (ρ, u, T) can be calculated by the mentioned variables, with the following formula;

Flow density: $\rho = \sum_i f_i$ (8)

Momentum: $\rho u_j = \sum_i f_i c_{ij}$ (9)

Temperature: $\rho RT = \sum_i g_i$ (10)

c) Large Eddy Simulation method

In this model the main aim is obtaining ν_t and $\alpha_t = (\frac{\nu_t}{\mathrm{Pr}_t})$ where Pr_t is turbulent Prandtle number which is assumed to be 0.5. In order to evaluate, we perform as follows:

$$\nu_t = (C\Delta)^2 \left(\left|\overline{S}\right|^2 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t} \nabla T . \frac{\vec{g}}{\left|\vec{g}\right|} \right)^{1/2} \tag{11}$$

where C is considered as Smagorinsky constant and in this paper it is assumed to be 0.1 [21] and Δ is gained from $\Delta = \sqrt{(\Delta x)^2 + (\Delta y)^2}$, and Δx and Δy are grid extents in X and Y directions.
For $\left|\overline{S}\right|$, we have:

$$\left|\overline{S}\right| = \sqrt{2\overline{S}_{\alpha\beta}\overline{S}_{\alpha\beta}} \tag{12}$$

$$\overline{S}_{\alpha\beta} = \left(\partial_\alpha \overline{u}_\beta + \partial_\beta \overline{u}_\alpha \right) / 2 \tag{13}$$

d) Lattice Boltzmann method based on Large Eddy Simulation model

Large eddy model is easily applied in Lattice Boltzmann method the way ν_t affects relaxation time [22-24].

$$\nu_{total} = c_s^2(\tau_\nu - 0.5) = \nu_0 + \nu_t \tag{14}$$

Where ν_{total} and ν_0 are total viscosity and initial viscosity respectively.

$$\tau_\nu = \frac{(\nu_0 + \nu_t)}{c_s^2} + 0.5 = \frac{\nu_0}{c_s^2} + 0.5 + \frac{\nu_t}{c_s^2} = \tau_0 + \frac{\nu_t}{c_s^2} \tag{15}$$

To obtain ν_t in Lattice Boltzmann method we have:

$$\left|\overline{S}\right| = \frac{3}{2\tau_m} \left| Q \right| \tag{16}$$

$$Q = \sum_{i=0}^{8} e_{i\alpha} e_{i\beta} (f_i - f_i^{eq}) \tag{17}$$

If we put $\left|\overline{S}\right|$ in Eq (11):

$$\nu_t = (C\Delta)^2 \left(\frac{9}{4\tau_m^2} \left| Q \right|^2 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t} \nabla T . \frac{\vec{g}}{\left|\vec{g}\right|} \right)^{1/2} \tag{18}$$

And if we substitute the above equation in Eq. (15):

$$\tau_{total} = \tau_0 + \frac{(C\Delta)^2 \left(\frac{9}{4\tau_m^2} |Q|^2 + \frac{\Pr}{\Pr_t} \nabla T . \frac{\vec{g}}{|\vec{g}|} \right)^{1/2}}{c_s^2} \tag{19}$$

To obtain relaxation time in temperature function equation we have:

$$\tau_c = \tau_{D0} + \frac{\alpha_t}{c_s^{\ 2}} = \tau_{D0} + \frac{\nu_t / \Pr_t}{c_s^{\ 2}} \tag{20}$$

Where $\tau_{D0} = \frac{\alpha_0}{c_s^{\ 2}} + 0.5$

Substituting new relaxation time in Eqs. (1) and (2) yields to Lattice Boltzmann equations based on large eddy model.

3. BOUNDARY CONDITIONS

a) Flow

The geometry of the present problem is shown in Fig. 1a. Implementation of boundary conditions is very important for the simulation. The unknown distribution functions pointing to the fluid zone at the boundaries nodes must be specified. Concerning the no-slip boundary condition, bounce back boundary condition is used on the solid boundaries. For instance, the unknown density distribution functions at the right wall can be determined by the following conditions:

$$f_{6,n} = f_{8,n} \quad , \quad f_{7,n} = f_{5,n} \quad , \quad f_{3,n} = f_{1,n} \tag{21}$$

where n indicates the lattice on the boundary.

b) Temperature

The upper and lower boundaries are adiabatic, so bounce back boundary condition is used on them. Temperature at the left and right walls are known, at the left wall T_H =1.0 and at the right wall T_C=0. Since we are using D2Q9, the unknown distribution functions are g_1, g_5 and g_8 at the left wall, and are evaluated as follows:

$$g_1 = T_H(\omega_1 + \omega_3) - g_3 \tag{22.1}$$

$$g_5 = T_H(\omega_5 + \omega_7) - g_7 \tag{22.2}$$

$$g_8 = T_H(\omega_8 + \omega_6) - g_6 \tag{22.3}$$

And for the right wall the unknown distribution functions are evaluated as follows:

$$g_3 = T_C(\omega_1 + \omega_3) - g_1 \tag{23.1}$$

$$g_7 = T_C(\omega_5 + \omega_7) - g_5 \tag{23.2}$$

$$g_6 = T_C(\omega_8 + \omega_6) - g_8 \tag{23.3}$$

Nusselt number *Nu* is one of the most important dimensionless parameters in describing the convective heat transport. The local Nusselt number and the average value at the hot and cold walls are calculated as follows:

$$NU_y = -\frac{L}{\Delta T}\frac{\partial T}{\partial x} \tag{24}$$

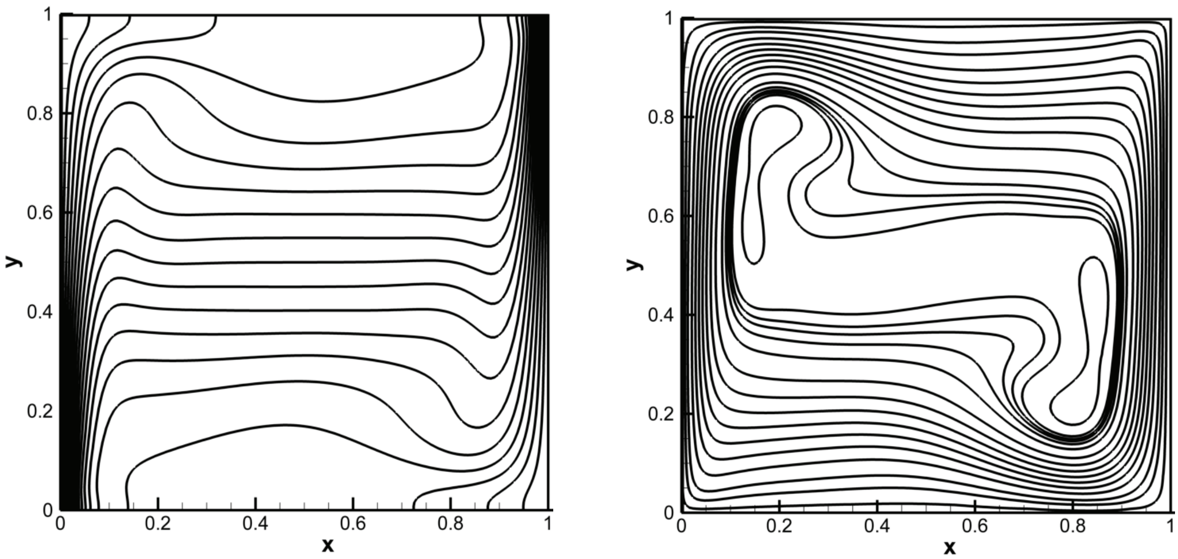
$$NU_{avg} = \frac{1}{L}\int_0^L NU_y dy \tag{25}$$

Finally, the following criterion to check for the steady-state solution is used;

$$Error = \max\left|T^{n+1} - T^{n}\right| \le 10^{-5} \tag{26}$$

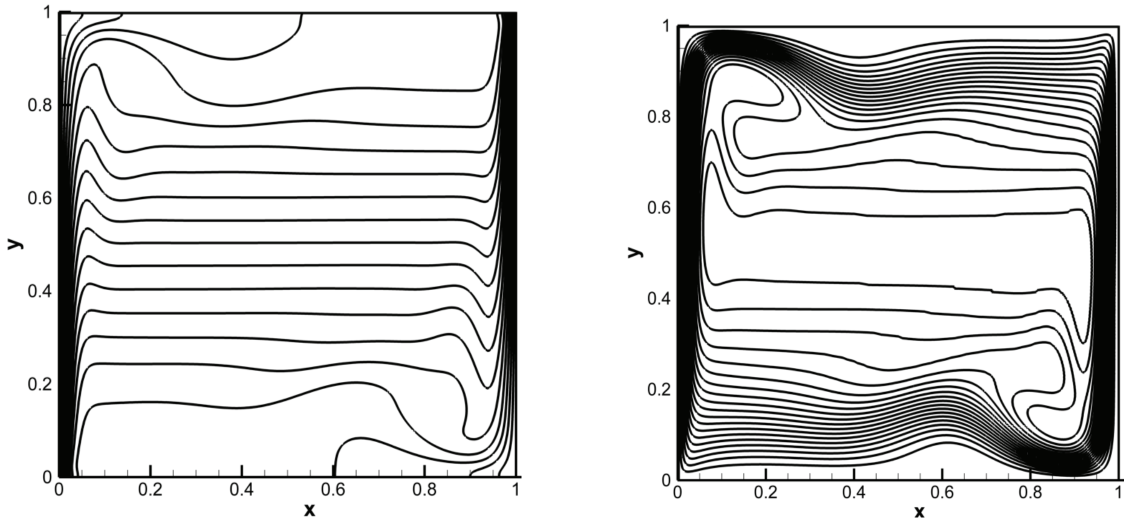
4. RESULTS AND DISCUSSION

In this section the results obtained in this study are represented. Figures 2-5 show temperature contour and stream lines for Rayleigh numbers 10^6 – 10^9. As is clear, the results are in a good agreement with other methods for numerically analyzing turbulent flow in a cavity. From 10^6 – 10^8 the isotherm behaves linearly close to the side walls. But at Ra=10^9 the isotherms have a crinkle state that can be due to the turbulence of the flow. The stream functions become dense as Rayleigh number increase from 10^6 – 10^7. On the other hand, with the increment of the Rayleigh number, some vortices appear at the cavity, whereas at Ra=10^9 the stream functions are combined from separate vortices throughout the cavity where Eddy formation in the upper region of the cavity is more than the lower region. This turbulence also leads to not having symmetric flow, unlike low Rayleigh numbers. To have a sufficient comprehension of temperature distribution in the cavity, a dimensionless temperature in the middle of the cavity is demonstrated in Fig 6. Table 1 shows the comparison of the average Nusselt numbers for different Rayleigh numbers. It is clearly seen that the results match with those of the previous works.



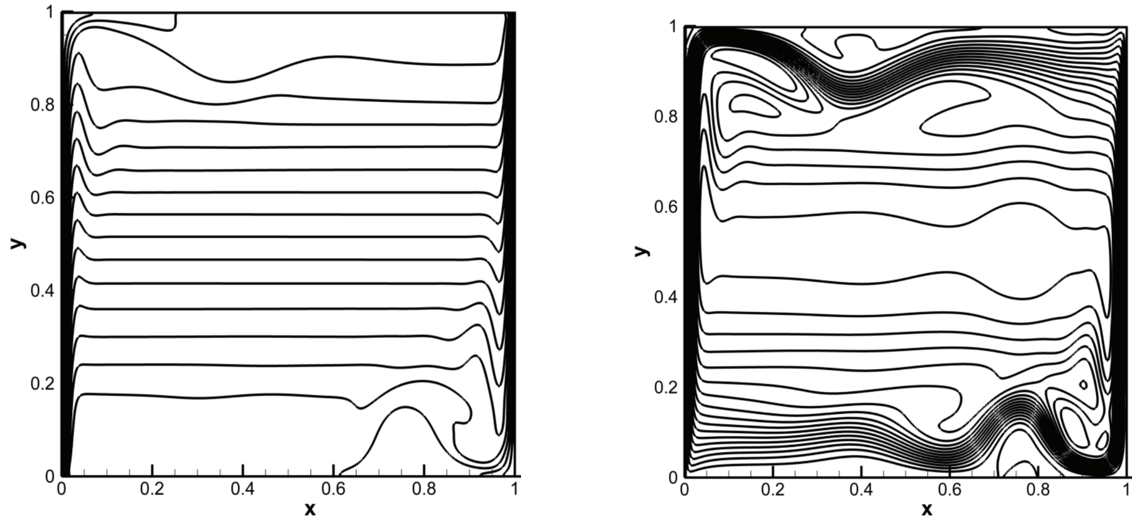
(a) (b)

Fig. 2. Isotherm (a) Streamline (b) for Ra=10^6



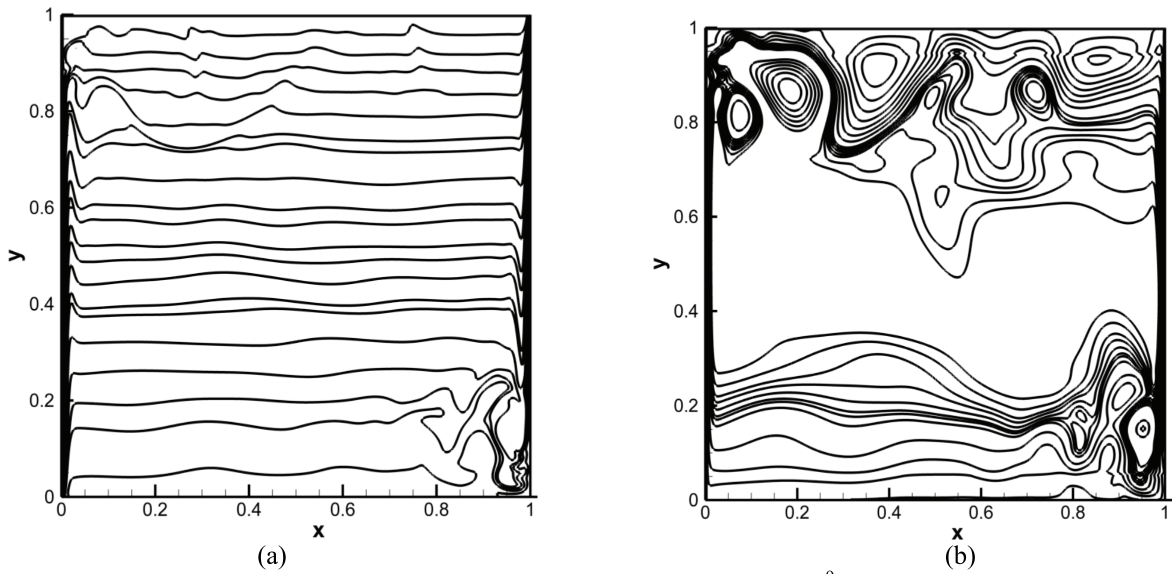
(a) (b)

Fig. 3. Isotherm (a) Streamline (b) for $Ra=10^7$



(a) (b)

Fig. 4. Isotherm (a) Streamline (b) for $Ra=10^8$



(a) (b)

Fig. 5. Isotherm (a) Streamline (b) for $Ra=10^9$

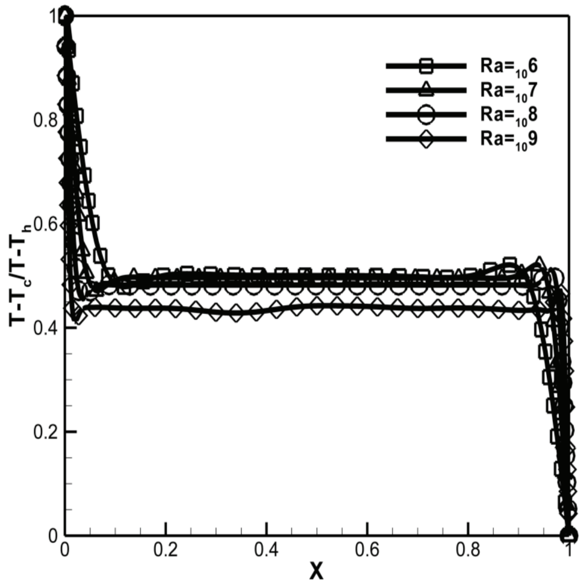


Fig. 6. Temperature distribution at mid-height of cavity for different Ra-values

Table 1. Comparison of average Nusselt number with previous works

Ra number	Mesh	Average Nu(Present)	Average Nu [19]	Average Nu[20]
10^6	128*128	8.7	8.8	8.65
10^7	256*256	17.2	__	16.8
10^8	512*512	31.2	32.3	30.5
10^9	1024*1024	58.1	60.1	57.4

Figure 7 illustrates distribution of velocity at y=0.5. It can be seen that from Ra=10^6 – 10^8 the trends are similar, but at Ra=10^9 a sinusoidal behavior exists that can be due to the formed vortices at the cavity. Figure 8 shows that in high Rayleigh numbers, the Nusselt number for the upper parts of the hot wall and the lower parts of the cool wall fluctuates, and it can be comprehended according to Fig. 5. As portrayed in Fig. 5, more Eddy currents are formed in the upper parts of the hot wall and lower parts of the cool wall, and this is the reason for instability of the flow in these regions.

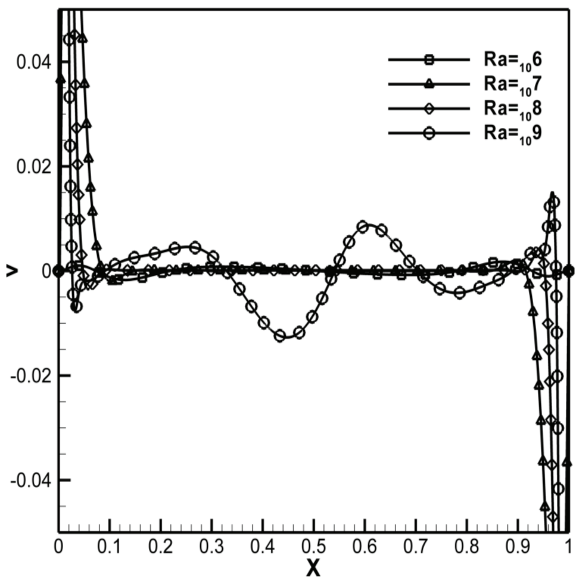


Fig. 7. Distribution of v-velocity component at mid-height of cavity for different Ra-values

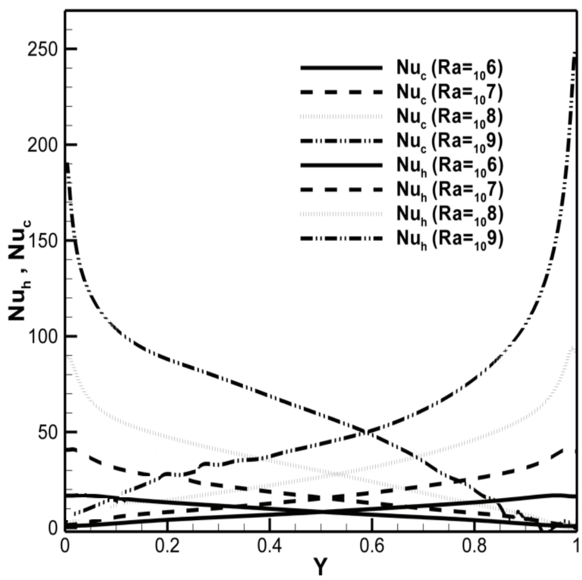


Fig. 8. Local Nusselt number distributions on the hot and cold walls at different Rayleigh numbers

5. CONCLUSION

Although the Lattice Boltzmann method is used repeatedly for solving laminar flow in square cavity, turbulent flow is not investigated as much. In this paper large eddy model is used to solve turbulent flow by Lattice Boltzmann method and also equilibrium distribution function is applied for temperature. The undertaken fluid is air with a Prandtl number of 0.71, and Rayleigh number is considered to be between 10^6 and 10^9. The results show that this method is in complete agreement with other methods for turbulent flow. As presented in the results, the increase of the Rayleigh number leads to an increase of turbulence in flow, and consequently the average Nusselt number increases.

NOMENCLATURE

f_α	distribution function for velocity field	ν	kinematic viscosity
g_α	distribution function for temperature field	τ_h	relaxation time for temperature field
g_α^{eq}	equilibrium distribution function for temperature field	α	thermal diffusivity
f_α^{eq}	equilibrium distribution function for velocity field	T	temperature
τ_m	relaxation time for velocity field	ν_t	turbulent eddy viscosity
ρ	density	$\Pr_t$	turbulent Prandtl number
w_α	weights for the particle equilibrium distribution weight	$\overline{S}_{\alpha\beta}$	strain rate tensor
e_α	discrete velocity		

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